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Deterministic: All data is known beforehand

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Probabilistic: Element of chance is involved

- You know the likelihood that something will happen, but you don't know when it will happen.
- Example. Roll a die until it comes up '5'.
  - Know that in each roll, a '5' will come up with probability $1/6$.
  - Don't know exactly when, but we can predict well.

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Example. The roll of the die . . . [is '5'] or [is odd] or [is prime] . . .

Example. $p(E_1) =$ ______, $p(E_2) =$ ______, $p(E_3) =$ ______.

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- **Subjective guess method:**
  If neither method above applies, give it your best guess.
  Example. How likely is it that your friend will come to a party?

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Example. You wake up and don't know what day it is.

- **Event 1:** Today is a weekday. E_1 vs. E_2
- **Event 2:** Today is cloudy. E_2 vs. E_3
- **Event 3:** Today is Modeling day. E_1 vs. E_3

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Example. What is the probability that you roll a blue 1 OR a red 6?

This does not work with *dependent* events.

Decision Trees

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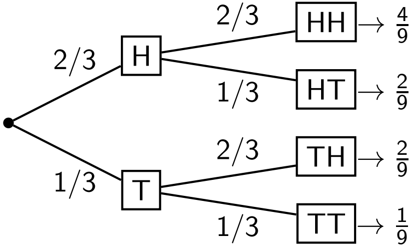

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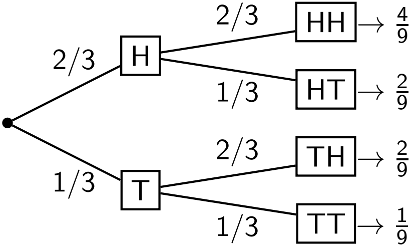

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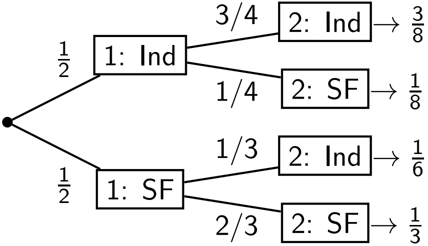
Example. Indiana and SF State U. play two soccer games. (p. 382)



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Definition: The **expected value** or **mean** of a random variable is the sum of the numbers weighted by their probabilities. Mathematically,

$$\mu = \mathbb{E}[X] = p(x_1)r(x_1) + p(x_2)r(x_2) + \cdots + p(x_n)r(x_n).$$

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Question: Interpretation of this number $\mathbb{E}[X]$?

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Component Reliability

Many systems consist of components pieced together.

Definition: The **reliability** of a system is its probability of success.

To calculate system reliability, first determine how reliable each component is; then apply rules from probability.

Example. Launch the space shuttle into space with a three-stage rocket.

$$\boxed{\text{Stage 1}} \rightarrow \boxed{\text{Stage 2}} \rightarrow \boxed{\text{Stage 3}}$$

⋆ In order for the rocket to launch, ______________________ ⋆

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Let $R_1 = 90\%$, $R_2 = 95\%$, $R_3 = 96\%$ be the reliabilities of Stages 1–3.

$p(\text{system success}) = p(\text{S1 success and S2 success and S3 success})$

Component Reliability

Example. Communicating with the space shuttle.

There are two independent methods in which earth can communicate with the space shuttle

- A microwave radio with reliability $R_1 = 0.95$
- An FM radio, with reliability $R_2 = 0.96$.

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$p(\text{system success}) = p(\text{MW radio success or FM radio success})$