

# Course Notes

Mathematical Models, Fall 2019

Queens College, Math 245

Prof. Christopher Hanusa

<http://qcpages.qc.cuny.edu/~chanusa/courses/245/19/>

# What is mathematical modeling?

Is it....

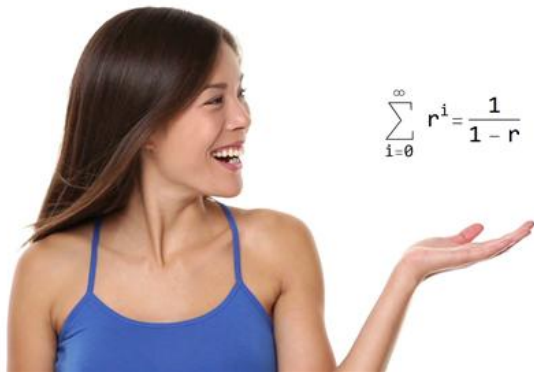
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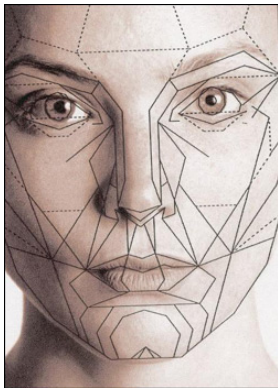
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No, that's modeling mathematics.

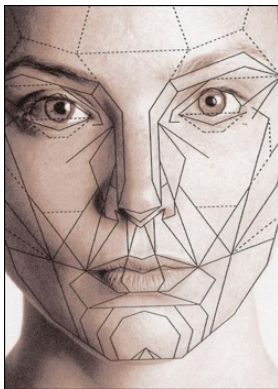
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No, that's the mathematics of modeling.

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No, that's **modeling** mathematical jewelry.



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No, that's a **model airplane**.  
But we're getting closer.

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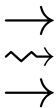
We will see: **Math is Everywhere.**

# A justification

**Why** do we model?

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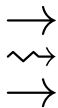
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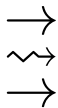
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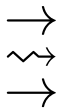
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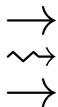
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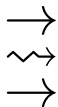
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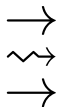
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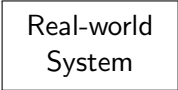
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**In this class:** We are going to create computer simulations in Python and use the language of mathematics to model the real world.

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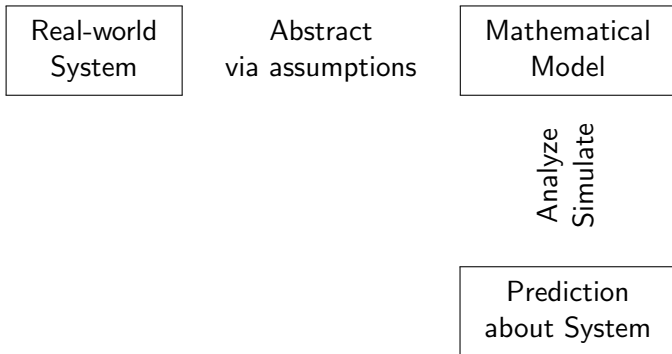
Abstract  
via assumptions

Mathematical  
Model

- Identify the most important variables in a real-world system

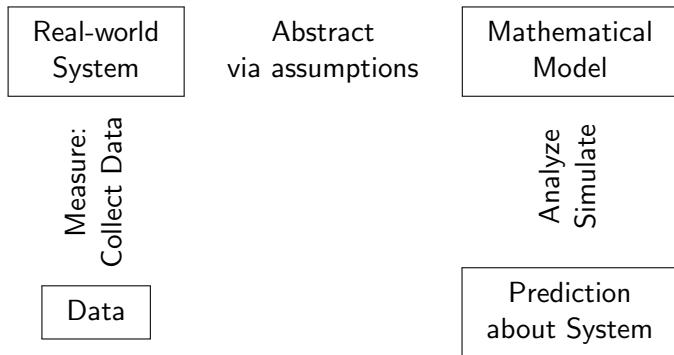


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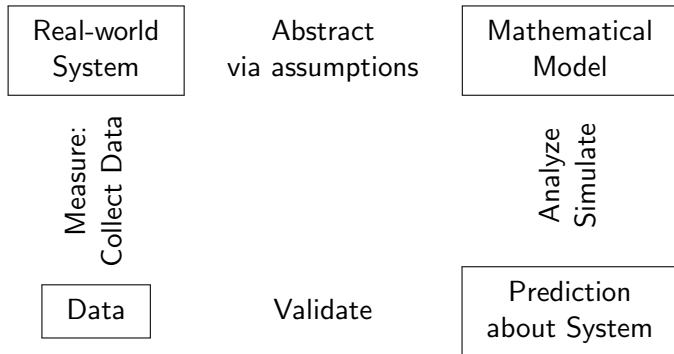
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- ▶ Analyze the model / Create a computer simulation
- ▶ Collect some data from the real world system
- ▶ Validate your model and refine / revise!

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- ▶ Learning after class
  - ▶ Finish tutorials, review notes, project work

## A normal day in this class

Outside  
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- ▶ Preparing for class
  - ▶ Do homework, pre-reading, daily question, prepare questions.

In class

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Everything posted online; first one (many parts) due Wednesday.



## Homework Notebook

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- ▶ Label pages with the date and label each question.

**Question 1-1.** (problem statement here)

Answer the question in complete sentences.

(Leave some space for notes from discussion.)

**Question 1-2.** (problem statement here)

⋮

## Meet the modelers

**Group Activity.** Get into groups of 3–4 people, with people you don't know. Take some time to get to know them:

- ▶ Introduce yourself. (your name, where you're from, your major)
- ▶ Fill out **the blank side of** your notecard:
  - ▶ Write your name. (Stylize if you wish.)
  - ▶ Write a few words about your name to help me remember.
  - ▶ *Draw* something in the remaining space.
- ▶ Discuss with your groupmates why you wrote what you wrote.
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- ▶ *Small talk suggestion:* What kept you busy this winter break?

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**Thought Question.** There are mathematical models used everyday in the real world. Brainstorm as many as you can. How do they impact your life?

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## Let's get Started!

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- ▶ Go to our course webpage:  
`qcpages.qc.cuny.edu/~chanusa/courses/245/19/`
- ▶ Find the day's plan > Content.
- ▶ Head to > Software.
- ▶ Use your Office365 account to access Azure Notebooks.
- ▶ Import the python notebooks from GitHub.
- ▶ While they are importing, access Google Classroom.
- ▶ Take a minute to answer the Daily Question.

## Jupyter notebook advice

Jupyter is the notebook environment. Python is the progr. language.

- ▶ **Always work in the code directory.**
- ▶ Make a new copy of the notebook before any modifications.
- ▶ Each time we start the server, all previous definitions are lost.
- ▶ Use the Python 3 kernel, not the Python 3.6 kernel.
- ▶ Jupyter notebooks look linear. They are not.
- ▶ Always evaluate the cells in order from top to bottom.

Let's flip a coin!

- ▶ We are using the `modsim` package; it must be imported each time we open the notebook.
- ▶ `modsim` relies on the `pint` package, so load it first.