# Evaluation of Mathematical Models

In what ways can a model be "good"? A model can be...

- **Accurate**
  - Is the output of the model very near to correct?
- **Descriptively Realistic**
  - Is the model based on assumptions which are correct?
- **Precise**
  - Are the predictors of the model definite numbers?
- **Robust**
  - Is the model relatively immune to errors in the input data?
- **General**
  - Does the model apply to a wide variety of situations?
- **Fruitful**
  - Are the conclusions useful?
  - Does the model inspire other good models?

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

This year, there are 10 million people between 18–22 years old. ($P$)

This year, there are 5 million students. ($S$)

We might create a model that says ____________________________.

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

This year, there are 10 million people between 18–22 years old. ($P$)

This year, there are 5 million students. ($S$)

We might create a model that says ______________________________.

Model Assumption 1:

Model Assumption 2:

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

This year, there are 10 million people between 18–22 years old. $(P)$

This year, there are 5 million students. $(S)$

We might create a model that says ______________________.

Model Assumption 1: Each college student is in 18–22 year old range.

Model Assumption 2:

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

This year, there are 10 million people between 18–22 years old. ($P$)

This year, there are 5 million students. ($S$)

We might create a model that says ______________________________.

Model Assumption 1: Each college student is in 18–22 year old range.

Model Assumption 2: One of every two is enrolled in college.

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

This year, there are 10 million people between 18–22 years old. ($P$)

This year, there are 5 million students. ($S$)

We might create a model that says ________________________.

Model Assumption 1: Each college student is in 18–22 year old range.

Model Assumption 2: One of every two is enrolled in college.

If next year we project 11,000,000 18–22 year olds, we would estimate the college population to be of size $E =$________________.

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

This year, there are 10 million people between 18–22 years old. ($P$)

This year, there are 5 million students. ($S$)

We might create a model that says ____________________________.

Model Assumption 1: Each college student is in 18–22 year old range.

Model Assumption 2: One of every two is enrolled in college.

If next year we project 11,000,000 18–22 year olds, we would estimate the college population to be of size $E =$__________________.

If this value is close to correct, we say our model is accurate. Otherwise, the model is **inaccurate**.

Problem:

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

This year, there are 10 million people between 18–22 years old. ($P$)

This year, there are 5 million students. ($S$)

We might create a model that says ____________________.

Model Assumption 1: Each college student is in 18–22 year old range.

Model Assumption 2: One of every two is enrolled in college.

If next year we project 11,000,000 18–22 year olds, we would estimate the college population to be of size $E =$ ________________.

If this value is close to correct, we say our model is accurate. Otherwise, the model is **inaccurate**.

Problem: (We won't whether we are accurate until next year!)

# Accuracy



*Definition:* A model is **accurate** if the answers it gives are correct.

Goal: Give a projection for how many students in the US.

This year, there are 10 million people between 18–22 years old. ($P$)

This year, there are 5 million students. ($S$)

We might create a model that says ____________________________.

Model Assumption 1: Each college student is in 18–22 year old range.

Model Assumption 2: One of every two is enrolled in college.

If next year we project 11,000,000 18–22 year olds, we would estimate the college population to be of size $E =$ ______________________.

If this value is close to correct, we say our model is accurate. Otherwise, the model is **inaccurate**.

Problem: (We won't whether we are accurate until next year!)

Question: How realistic is this model?

# Descriptively Realistic

Accurate
**Real**
Precise
Robust
General
Fruitful

*Definition:* A mathematical model is **descriptively realistic** if the model is based on correct and realistically verifiable assumptions.

# Descriptively Realistic

Accurate
**Real**
Precise
Robust
General
Fruitful

*Definition:* A mathematical model is **descriptively realistic** if the model is based on correct and realistically verifiable assumptions.

Improvement: Incorporate other age groups!

# Descriptively Realistic

Accurate
**Real**
Precise
Robust
General
Fruitful

*Definition:* A mathematical model is **descriptively realistic** if the model is based on correct and realistically verifiable assumptions.

Improvement: Incorporate other age groups!

Assumption 3: College students are either:

- 18–22 ($P_a$ of these)
- 23 or older ($P_b$ of these)
- 17 or younger ($P_c$ of these)

Assumption 4: The enrolled percentages for each age range is:

- 30% for people aged 18–22
- 3% for people aged 23 or older
- 1% for people aged 17 or younger

# Descriptively Realistic



*Definition:* A mathematical model is **descriptively realistic** if the model is based on correct and realistically verifiable assumptions.

Improvement: Incorporate other age groups!

Assumption 3: College students are either:

- 18–22 ($P_a$ of these)
- 23 or older ($P_b$ of these)
- 17 or younger ($P_c$ of these)

Assumption 4: The enrolled percentages for each age range is:

- 30% for people aged 18–22
- 3% for people aged 23 or older
- 1% for people aged 17 or younger

We would estimate the college population to be of size

$$E = 0.3P_a + 0.03P_b + 0.01P_c.$$

# Precision



A model is **precise** if the prediction is
- a definite number
- a definite function
- etc.

# Precision



A model is:

- **precise** if the prediction is
  - a definite number
  - a definite function
  - etc.
- **imprecise** if the prediction is
  - a range of numbers
  - a set of functions
  - etc.

# Precision



A model is:

- **precise** if the prediction is
  - a definite number
  - a definite function
  - etc.
- **imprecise** if the prediction is
  - a range of numbers
  - a set of functions
  - etc.

*Question:* Are the enrollment models precise or imprecise? Why?

# Precision

Accurate
Real
**Precise**
Robust
General
Fruitful

A model is

- **precise** if the prediction is
  - a definite number
  - a definite function
  - etc.
- **imprecise** if the prediction is
  - a range of numbers
  - a set of functions
  - etc.

*Question:* Are the enrollment models precise or imprecise? Why?

Assumption 1: Each college student is in 18–22 year old range.

Assumption 2*: The percentage of 18–22 in college is 46%–50%.

# Precision



A model is

- **precise** if the prediction is
  - a definite number
  - a definite function
  - etc.
- **imprecise** if the prediction is
  - a range of numbers
  - a set of functions
  - etc.

*Question:* Are the enrollment models precise or imprecise? Why?

Assumption 1: Each college student is in 18–22 year old range.

Assumption 2*: The percentage of 18–22 in college is 46%–50%.

Model Conclusion:
$$(0.46)(11,000,000) \leq E \leq (0.5)(11,000,000)$$
$$5,060,000 \leq E \leq 5,500,000.$$

# Precision



A model is

- **precise** if the prediction is
  - a definite number
  - a definite function
  - etc.
- **imprecise** if the prediction is
  - a range of numbers
  - a set of functions
  - etc.

*Question:* Are the enrollment models precise or imprecise? Why?

Assumption 1: Each college student is in 18–22 year old range.

Assumption 2*: The percentage of 18–22 in college is 46%–50%.

Model Conclusion:

$$(0.46)(11,000,000) \leq E \leq (0.5)(11,000,000)$$
$$5,060,000 \leq E \leq 5,500,000.$$

Is this model precise or imprecise?

# Precision



A model is

- **precise** if the prediction is
  - a definite number
  - a definite function
  - etc.
- **imprecise** if the prediction is
  - a range of numbers
  - a set of functions
  - etc.

*Question:* Are the enrollment models precise or imprecise? Why?

Assumption 1: Each college student is in 18–22 year old range.

Assumption 2*: The percentage of 18–22 in college is 46%–50%.

Model Conclusion:
$$(0.46)(11,000,000) \leq E \leq (0.5)(11,000,000)$$
$$5,060,000 \leq E \leq 5,500,000.$$

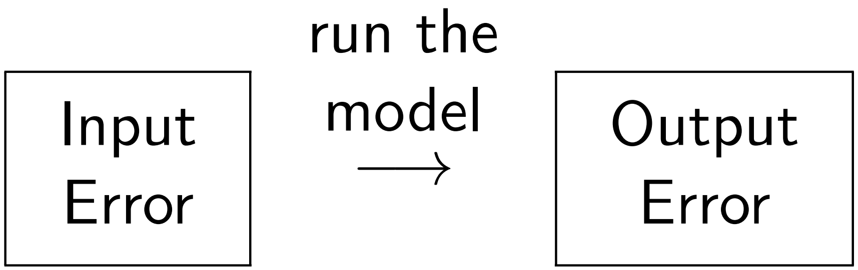
Is this model precise or imprecise?

Is this model perhaps more helpful?

# Robustness



*Definition:* A model is **robust** if it is relatively immune to errors in the input data.



# Robustness



*Definition:* A model is **robust** if it is relatively immune to errors in the input data.

Input Error $\xrightarrow{\text{run the model}}$ Output Error

Example. If our population estimate (input) has an error of 10%, how much does our college enrollment estimate (output) change?

# Robustness



*Definition:* A model is **robust** if it is relatively immune to errors in the input data.

Input Error —(run the model)→ Output Error

Example. If our population estimate (input) has an error of 10%, how much does our college enrollment estimate (output) change?

Ask: Is the output error less than 10% or more than 10%?

# Robustness



*Definition:* A model is **robust** if it is relatively immune to errors in the input data.

Input Error → (run the model) → Output Error

Example. If our population estimate (input) has an error of 10%, how much does our college enrollment estimate (output) change?

Ask: Is the output error less than 10% or more than 10%?

- Some models **magnify** the errors that exist in the input data; we say these models are **sensitive to error** or **not robust**.

# Robustness



*Definition:* A model is **robust** if it is relatively immune to errors in the input data.

Input Error $\xrightarrow{\text{run the model}}$ Output Error

Example. If our population estimate (input) has an error of 10%, how much does our college enrollment estimate (output) change?

Ask: Is the output error less than 10% or more than 10%?

- Some models **magnify** the errors that exist in the input data; we say these models are **sensitive to error** or **not robust**.

*Question:* What does 10% error mean?

# Generality



*Definition:* A model is **general** if it applies to a variety of situations.

*Question:* Where does our population model apply?

*Question:* How can we make our model more general?

Accurate
Real
Precise
Robust
General
**Fruitful**

# Fruitfulness

*Definition:* A model is **fruitful** if either

- Its conclusions are useful.
- It inspires other good models.

# Fruitfulness

Accurate
Real
Precise
Robust
General
**Fruitful**

*Definition:* A model is **fruitful** if either

- Its conclusions are useful.
- It inspires other good models.

Our college enrollment model is fruitful in multiple ways:

- Planning for demand for educational grants, dormitory space, teacher hiring, etc.
- The ideas we implemented are transferrable to other situations.

# Fruitfulness

Accurate
Real
Precise
Robust
General
**Fruitful**

*Definition:* A model is **fruitful** if either

- Its conclusions are useful.
- It inspires other good models.

Our college enrollment model is fruitful in multiple ways:

- Planning for demand for educational grants, dormitory space, teacher hiring, etc.
- The ideas we implemented are transferrable to other situations.

Example. How many automobiles would be junked in a given year?

- Cars play the role of people.
- Partitioning by age of cars gives better results

# The Advantage of Inaccuracy

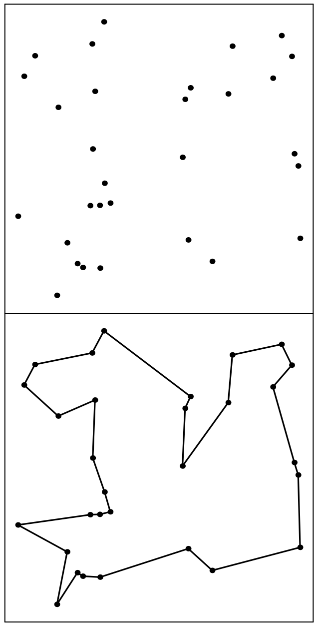
**Accurate**
Real
Precise
Robust
General
Fruitful

Often accuracy is very expensive! (computationally / financially).

# The Advantage of Inaccuracy

**Accurate**
Real
Precise
Robust
General
Fruitful

Often accuracy is very expensive! (computationally / financially).



Example. The Traveling Salesman Problem

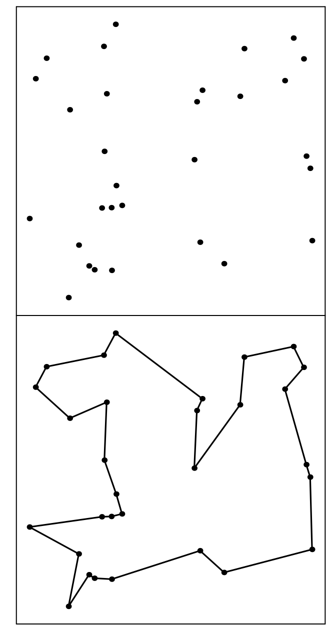
**TSP**: Given: Home & Set of Destinations,
Find the shortest path starting and ending at home, visiting each place once along the way.

# The Advantage of Inaccuracy

**Accurate**
Real
Precise
Robust
General
Fruitful

Often accuracy is very expensive! (computationally / financially).

Example. The Traveling Salesman Problem



**TSP**: Given: Home & Set of Destinations,
Find the shortest path starting and ending at home, visiting each place once along the way.

With many locations, there are (inexpensive and inaccurate) or (expensive and accurate) algorithms to solve these problems.

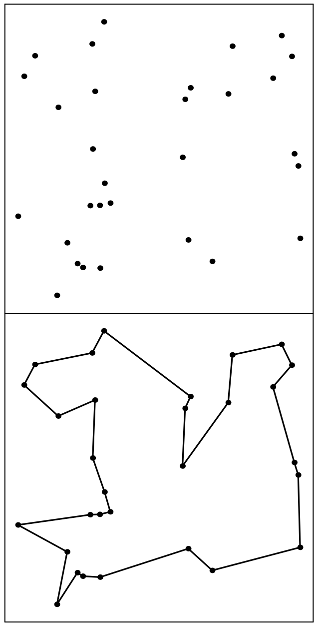
# The Advantage of Inaccuracy

**Accurate**
Real
Precise
Robust
General
Fruitful

Often accuracy is very expensive! (computationally / financially).

Example. The Traveling Salesman Problem

**TSP**: Given: Home & Set of Destinations,
Find the shortest path starting and ending at home, visiting each place once along the way.



With many locations, there are (inexpensive and inaccurate) or (expensive and accurate) algorithms to solve these problems.

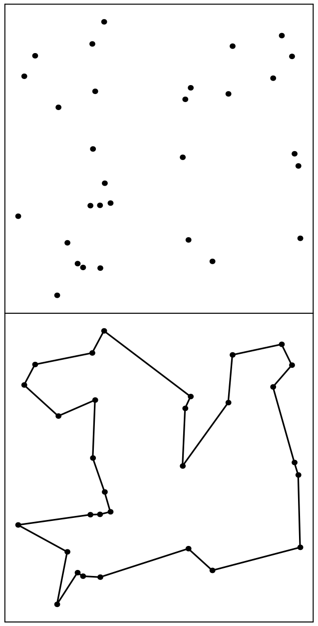
Your approach will depend on the particular application and your scale:

# The Advantage of Inaccuracy

**Accurate**
Real
Precise
Robust
General
Fruitful

Often accuracy is very expensive! (computationally / financially).

Example. The Traveling Salesman Problem



**TSP**: Given: Home & Set of Destinations,
Find the shortest path starting and ending at home, visiting each place once along the way.

With many locations, there are (inexpensive and inaccurate) or (expensive and accurate) algorithms to solve these problems.

Your approach will depend on the particular application and your scale:

- If you visit the same places every day, run the expensive model **once initially** in order to save money in the long run.

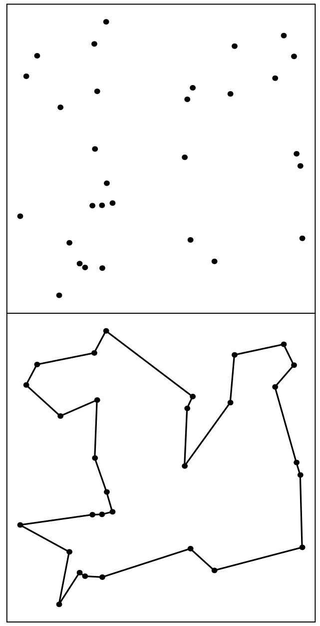
**Accurate**
Real
Precise
Robust
General
Fruitful

# The Advantage of Inaccuracy

Often accuracy is very expensive! (computationally / financially).

Example. The Traveling Salesman Problem

**TSP**: Given: Home & Set of Destinations,
Find the shortest path starting and ending at home, visiting each place once along the way.



With many locations, there are (inexpensive and inaccurate) or (expensive and accurate) algorithms to solve these problems.

Your approach will depend on the particular application and your scale:

- If you visit the same places every day, run the expensive model **once initially** in order to save money in the long run.
- If you visit different places every day, run the inexpensive algorithm daily.

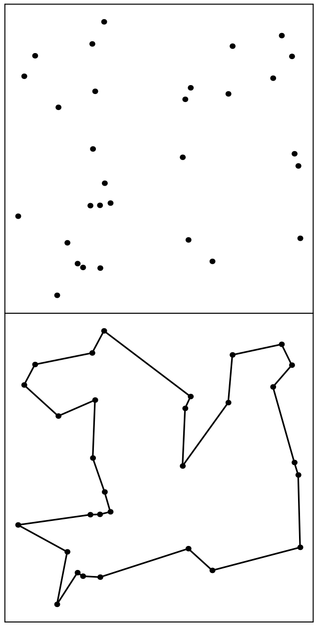
# The Advantage of Inaccuracy

**Accurate**
Real
Precise
Robust
General
Fruitful

Often accuracy is very expensive! (computationally / financially).

Example. The Traveling Salesman Problem

**TSP**: Given: Home & Set of Destinations,
Find the shortest path starting and ending at home, visiting each place once along the way.



With many locations, there are (inexpensive and inaccurate) or (expensive and accurate) algorithms to solve these problems.

Your approach will depend on the particular application and your scale:

- If you visit the same places every day, run the expensive model **once initially** in order to save money in the long run.
- If you visit different places every day, run the inexpensive algorithm daily. (Unless you're UPS or FedEx.)