1. (12 pts) Answer the following true or false questions in your blue book. There is no partial credit.
(a) $\mathbf{T}$ or $\mathbf{F}$ : In every cycle graph $C_{n}$, a maximal matching is always a maximum matching.
(b) $\mathbf{T}$ or $\mathbf{F}$ : There is a closed knight's tour on a $4 \times 5$ rectangular board.
(c) Let $G$ be a connected graph with degree sequence ( $7,5,4,4,4,4,2,2,2,2$ ).
$\mathbf{T}$ or $\mathbf{F}$ : It is possible to create a drawing of $G$ without picking up your pencil.
(d) $\mathbf{T}$ or $\mathbf{F}$ : The cube graph is self-dual.
2. ( 8 pts ) Determine the chromatic number of this graph $Q$ : [Hint: Use a known theorem, do not actually color $Q$.]
3. (10 pts) Show that the thickness of $K_{6}$ is 2 . Use an argument that involves a decomposition of $K_{6}$.

[Important: DO NOT use a formula.]
4. (10 pts) Let $N=(V, E)$ be a network with capacities $c_{e}$ on every edge $e \in E$. Suppose that $\varphi$ is a flow on $N$ and that for a subset $X \subset V,\left[X, X^{c}\right]$ is an st-cut in $N$. Explain precisely why the throughput of $\varphi$ must be less than or equal to the capacity of $\left[X, X^{c}\right]$.
[In this problem $\varphi$ is not necessarily a max flow, nor is $\left[X, X^{c}\right]$ a min cut.]
5. (10 pts) Use the Hungarian algorithm to find a maximum matching on the graph below, starting with the given matching.
[Feel free to use the extra copies of the graph that are provided.]

