

MATH 636, Fall 2015

HOMEWORK 13

To be prepared for presentation on Tuesday, November 17.

*Background reading: Combinatorics: A Guided Tour, Section 4.1 plus additional material.*

**Only** consult with your classmates or professor to discuss the problem set.

We will discuss solutions to these questions in class.

- 13-1.** Use a bijection to show that sequences  $1 \leq a_1 \leq a_2 \leq \cdots \leq a_n$  of length  $n$ , where each  $a_i \leq i$  are also counted by the Catalan number  $C_n$ . For example, when  $n = 3$ , the five sequences are 111, 112, 113, 122, and 123.

[*Hint: Look at the boxes to the left of a Dyck path.*]

- 13-2.** (a) Calculate  $\text{des}(\pi)$ ,  $\text{inv}(\pi)$ , and  $\text{maj}(\pi)$  for  $\pi = 963852741$ .  
(b) Let  $\pi$  be an  $n$ -permutation with reverse  $\hat{\pi}$ . How is  $\text{inv}(\pi)$  related to  $\text{inv}(\hat{\pi})$