MATH 636, Fall 2015
Homework 14
To be prepared for presentation on Thursday, November 19.

Background reading: Combinatorics: A Guided Tour, Section 4.1 plus additional material.

Only consult with your classmates or professor to discuss the problem set.

We will discuss solutions to these questions in class.

14-1. Two combinatorial interpretations of the q-binomial coefficients are given on page 124 of the course notes.

(a) Show that for the permutations π of the multiset $\{1^2, 2^3\}$, $\sum_{\pi \in S_{2,3}} q^{\mathsf{inv}(\pi)} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}_q$.

(b) Show that for the set of lattice paths P from $(0,0)$ to $(2,3)$, $\sum_{P \in \mathcal{P}} q^{\mathsf{area}(P)} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}_q$.

14-2. Let $\mathcal{C}_n$ denote the set of compositions of n.
For any composition c, define the statistic $\mathsf{parts}(c)$ to be the number of parts of c.
[*In other words, if c is the composition $c_1 + c_2 + \cdots + c_k$, then* $\mathsf{parts}(c) = k$.]

(a) Compute $f_n(q) = \displaystyle\sum_{c \in \mathcal{C}_n} q^{\mathsf{parts}(c)}$.

(b) Use your answer to part (a) to show directly $\displaystyle\lim_{q \to 1} f_n(q) = 2^{n-1}$.

[*Note: We expect part (b) to be true because we know there are 2^{n-1} compositions of n, and part (a) is constructing a q-analog.*]