MATH 636, Fall 2015
Homework 4
To be prepared for presentation on Thursday, September 17.

Background reading: Combinatorics: A Guided Tour, Section 1.3.

Only consult with your classmates or professor to discuss the problem set.

We will discuss solutions to these questions in class.

4-1. In the following problem, suppose $k \leq n$.

    (a) How many functions are there from $[k]$ to $[n]$?

    (b) How many bijections are there from $[k]$ to $[n]$?

    (c) How many one-to-one functions are there from $[k]$ to $[n]$?

    (d) How many onto functions from $[k]$ to $[n]$ are not one-to-one?

4-2. Create and prove a bijection between two-member subsets of $\{1, 2, ..., n, n+1\}$ and all possibilities of placing one pair of parentheses in a string of n letters. For example, when $n = 3$, we see that there are six two-member subsets of $\{1, 2, 3, 4\}$:

$$\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \text{ and } \{3, 4\}.$$

and there are six ways to place one pair of parentheses in the word *abc*:

$$(a)bc, \ a(b)c, \ ab(c), \ (ab)c, \ a(bc), \text{ and } (abc).$$

Notice that $a()bc$ is not valid—there are no letters between the parentheses.