

MATH 636, Fall 2015

HOMEWORK C

Due 3:10PM on Thursday, October 29.

**Assignment:** Turn in written or typed solutions to homework questions 8-1, 9-1 parts (d) and (e), 10-1, and Question C-1:

**C-1.** Revisit homework question 2-1. Set up a *multivariate* generating function

$$A(q, r) = \sum_{c \geq 0} \sum_{n \geq 0} a_{cn} q^c r^n$$

where  $a_{cn}$  is the number of ways in which  $c$  cents can be made using  $n$  coins. This generating function starts out:

$$A(q, r) = q^0 r^0 + q^1 r^1 + q^2 r^2 + q^3 r^3 + q^4 r^4 + q^5 (r + r^5) + q^6 (r^2 + r^6) + \dots$$

- (a) Determine the compact form of  $A(q, r)$ , which gives the number of ways to make change for any amount of money using only pennies, nickels, dimes, and quarters.
- (b) Use *Wolfram Alpha* or *Mathematica* to explore at the coefficients of this generating function.

[*Hint: You will have to expand the Taylor series twice, once in each variable.*]

- (c) Use your answer from part (b) to explain why the answer to the Question 2-1 is 39 cents, and find the smallest monetary amount that is expressible in exactly  $k$  with exactly 15 coins for  $3 \leq k \leq 6$ .

**Follow the posted homework guidelines** when completing this assignment.

Please **only** consult with your classmates or professor to discuss the problem set.

In particular, remember that you must **fully justify** any assertions you make.