MATH 636, Fall 2015
Practice Problems
in preparation for Exam 1 on October 13, 2015

The exam covers:

- *Combinatorics: A Guided Tour*, Sections 1.1–1.4, 2.1–2.4
- Additional topics that are included in the course notes, including Combinatorial proofs, the Principle of Inclusion/Exclusion, the Square-Domino interpretation of Fibonacci numbers.

Below are some questions that practice concepts from the class.

– Book questions: 1.2.6, 1.2.9, **1.2.18**, **1.3.5**, 1.3.12, 1.4.8, 2.1.3, 2.1.4, **2.1.12**, 2.1.13, 2.2.7cdf, 2.3.1, **2.3.4**, 2.3.6, 2.4.5, **2.4.11**, **3.1.10**, **3.1.17** (Bolded exercises are especially encouraged.)

– *Challenge questions:* 2.1.16, 2.2.11

P1. Write down a list of all combinatorial interpretations of $\binom{n}{k}$, $\left(\!\binom{n}{k}\!\right)$, and $(n)_k$ from the book.

P2. Use the square-domino interpretation of the Fibonacci numbers to prove that $f_{m+n} = f_m f_n + f_{m-1} f_{n-1}$ for positive integers m and n.

P3. A **composition** of a positive integer n is a way to write n as a sum of positive integers with no restrictions and where order matters. For example, the compositions of 3 are $1+1+1$, $2+1$, $1+2$, and 3 itself.

Let n be a fixed positive integer. Let $\mathcal{C}$ be the set of compositions of n, and let $\mathcal{B}$ be the set of binary words of length $n-1$. Find an explicit bijection between $\mathcal{C}$ and $\mathcal{B}$, and prove that it is a bijection.

P4. There are many games that have a combinatorial flavor. One of these games is *Tantrix*. (See http://www.tantrix.com/). *Tantrix* tiles have six sides (they're hexagonal); on each tile there are three colored paths (using three out of the four colors red, yellow, green, and blue). These paths leave one of the six sides of the tile and arrive at some other side of the tile. Determine the possible number of tiles that satisfy these criteria. [Hint: draw all possible ways that three pairs of two sides can be paired up. Then figure out in how many ways they can be colored (realizing that rotations may define some sort of equivalence class. Last, realize that the answer is not 56 (the number of tiles in a *Tantrix* set) because there are a few valid tiles that are not included.]