1. **Donut counting time!** You walk into a donut shop that has 15 different types of donuts, each with unlimited supply. **...mmmMMMmmm infinite donuts...**

   For each of the following scenarios, **write at least a sentence** to justify your answer!

   (a) (4 pts) In how many ways can you choose a sequence of five donuts to eat, eating each delicious donut right after the other in order?

   (b) (4 pts) In how many ways can you choose a dozen donuts to take home, given that you love each and every type of donut offered for sale?

   (c) (6 pts) You decide to buy one donut of each type and you take them to your study group of five friends (so that you are six in total). In how many ways can you allocate all the donuts to the group members, in which each group member must receive at least one donut?

   (d) (6 pts) You get to the store only 5 minutes before closing and the only donuts that remain are: 3 chocolate, 4 vanilla with sprinkles, and 10 old fashioned donuts. In how many ways can you construct a half-dozen donuts?

2. (10 pts) In two or more paragraphs, explain the process involved in using the equivalence principle to calculate the number of permutations of the word COMBINATORICS.

   [*No proofs are required, but I expect you to describe the entire setup and process.*]

3. (15 pts) Give a combinatorial proof of the identity

$$n! = \sum_{k=0}^{n} \binom{n}{k} D_k,$$

   where D_k denotes the number of *derangements* of k objects.

   [*You may buy a hint for the cost of 3 points.*]

4. (15 pts) The number of binary words of length n in which 1's only appear in even-length sequences are counted by the Fibonacci numbers. Prove that this is true by finding and proving a bijection with square-domino tilings of a $1 \times n$ board.

   [*As an example,* $f_5 = 8$ *because there are eight binary words of length* n *in which* 1*'s only appear in even length sequences:* 00000, 00011, 00110, 01100, 11000, 01111, 11011, *and* 11110.]

   [*You may buy a hint about the description of the bijection for the cost of 3 points.*]