Math 636 Exam 2
4 Dec 2014
60 points total

Show all work for full or partial credit.

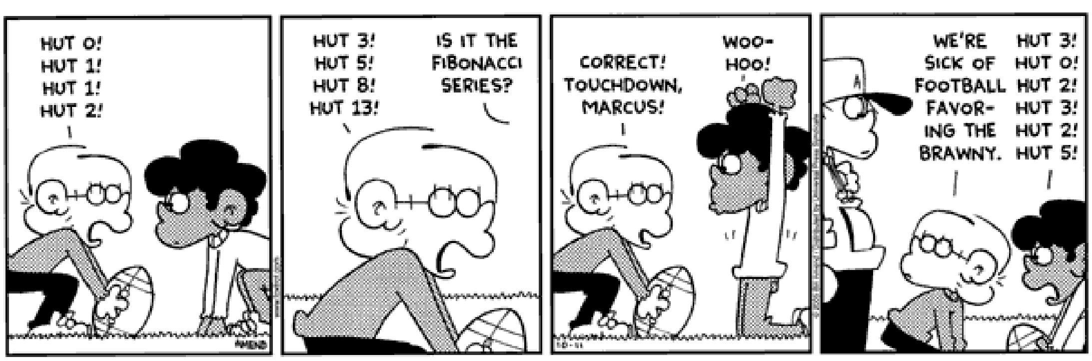
Name:____

Problem Scores

1	/10		4	/10
2	/15		5	/ 5
3	/10		6	/10

Total Score: /60
Exam Grade:
Projected Course Grade:

Do not spend too much time on any one problem.



1. (10 pts) In the Foxtrot comic above, Marcus starts listing the terms of a sequence ($s_0 = 3$, $s_1 = 0$, etc.) that satisfies the recurrence $s_n = s_{n-2} + s_{n-3}$. Determine a compact form for the generating function $\sum_{n\geq 0} s_n x^n$.

   [*Important: Do* ***NOT*** *find a formula for* s_n*.*]

(Question 1 is on the other side!)

2. (15 pts) (**QuAQ**'s) Quick answer questions.
   [*Explanations are not required, but may get you partial credit.*]

   (a) (5 pts) Solve for the coefficient of x^k of $f(x) = \dfrac{1}{4-x} + e^x$.

   (b) (5 pts) Give the compact form of the generating function for the number of partitions of n such that each part is a square number. (For example, $9\,4\,4\,1$ is OK but $8\,5\,4\,1$ is not.)

   (c) (5 pts) Suppose that λ is a partition of $n \geq 2$ with no parts of size 1. What must be true about λ^c, its conjugate partition?

3. (a) (5 pts) How many triangulations of a regular 8-gon are there? Write your answer as an expression involving a binomial coefficient.

   (b) (5 pts) Draw a triangulation of a regular 8-gon, and use a Catalan bijection from class to determine the corresponding multiplication scheme.

4. (10 pts) The generating function $\sum_{k\geq 0} C_k x^k$ for the Catalan numbers satisfies the functional equation

$$C(x) = 1 + x\,C(x)^2.$$

   (a) Use convolution to find an explicit expression for the coefficient of x^n of both sides of this equation.

   (b) Use the recurrence relation that arises by equating the coefficients in part (a) to show that the fourth Catalan number is $C_4 = 14$.

5. (5 pts) Give the precise definition of a combinatorial statistic.

6. (10 pts) Consider the set S of permutations of the multiset $\{1, 1, 2, 3\}$. Show that

$$\sum_{\pi \in S} q^{\mathsf{inv}(\pi)} = \begin{bmatrix} 4 \\ 2,1,1 \end{bmatrix}_q = \frac{[4]_q!}{[2]_q![1]_q![1]_q!}.$$