Name:

1	T
2	F
3	T
4	T
5	T
6	T
7	F
8	T
9	T
10	T

11.

True/False [1 pt each] For each of the following statements, decide whether it is true or false. Put **T** or **F** on the answer sheet.

1. Let the universal set be the set of digits $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. If $A = \{4, 3, 6, 7, 1, 9\}$ and $B = \{5, 6, 8, 4\}$ then $\overline{A} \cup B = \{0, 2, 4, 5, 6, 8\}$.

Answer. True. $\overline{A} = \{0, 2, 5, 8\}$ so $\overline{A} \cup B = \{0, 2, 5, 8, 6, 4\}$.

2. If $A = \{a, b, c, d, e\}$, $B = \{d, e, f\}$ and $C = \{1, 2, 3\}$, then

$$(B - A) \times C = \{(e, 1), (e, 2), (e, 3), (d, 3), (d, 2), (d, 1)\}.$$

Answer. False. $B - A = \{f\}$ so $B - A \times C = \{(f, 1), (f, 2), (f, 3)\}$.

3. $\{\{\emptyset\}\} \subseteq \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

Answer. True. Here, the statement is true if every element belonging to the set on the left also belongs to the set on the right. The set on the left contains only one element, namely $\{\emptyset\}$ which is the second element listed in the set on the right.

4. $|\{x \in \mathbb{Z} : |2x - 1| < 6\}| > |\mathcal{P}(\{2, 6\})|$

Answer. True. $\{x \in \mathbb{Z} : |2x - 1| < 6\} = \{-2, -1, 0, 1, 2, 3\}$ which has cardinality six. On the other side, we have $\mathcal{P}(\{2, 6\}) = \{\emptyset, \{2\}, \{6\}, \{2, 6\}\}$ which has cardinality four.

5. For each $n \in \mathbb{N}$, define a set $A_n \subseteq \mathbb{Z}$ by $A_n = \{-n, \ldots, -2, -1, 0, 1, 2, \ldots, n\}$. Then

$$\bigcup_{n \in \mathbb{N}} A_n = \mathbb{Z} \text{ and } \bigcap_{n \in \mathbb{N}} A_n = \{-1, 0, 1\}.$$

Answer. True.

$$\bigcup_{n \in \mathbb{N}} A_n = \{-1, 0, -1\} \cup \{-2, -1, 0, 1, 2\} \cup \{-3, -2, -1, 0, 1, 2, 3\} \cup \cdots = \mathbb{Z}$$

since every integer is contained in at least one of the A_n's. Also,

$$\bigcup_{n \in \mathbb{N}} A_n = \{-1, 0, -1\} \cap \{-2, -1, 0, 1, 2\} \cap \{-3, -2, -1, 0, 1, 2, 3\} \cap \cdots = \{-1, 0, 1\}$$

since $-1, 0, 1 \in A_n$ for every $n \in \mathbb{N}$ and there are no other integers in A_1.

6. Let P and Q be propositions. The propositions $P \Rightarrow Q$ and $(\sim P) \vee Q$ are logically equivalent.

Answer. True. Check the second and fourth columns of the truth table below:

P	Q	$P \Rightarrow Q$	$\sim P$	$(\sim P) \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

7. If P and Q are propositions that are true then $(P \vee Q) \wedge \sim (P \wedge Q)$ is also true.

Answer. False. Notice that in order for $(P \vee Q) \wedge \sim (P \wedge Q)$ to be true it is necessary that $\sim (P \wedge Q)$ be true. But if P and Q are both true, $P \wedge Q$ is true and $\sim (P \wedge Q)$ is false.

8. $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z}\ (x + y = 0)$

Answer. True. To prove it, let x be any integer. Choose $y = -x$ to see that $x + y = 0$.

9. $\exists y \in \mathbb{Z}\ \forall x \in \mathbb{Z}\ (x + y = x)$

Answer. True. To prove it, let $y = 0$. It is true that $x + 0 = x$ for any integer x.

10. For all sets A, B, C $(A \subseteq B \wedge A \subseteq C) \Rightarrow (A \cap B = A \cap C)$.

Answer. True. Notice that if $A \subseteq B$, then $A \cap B = A$. Also, if $A \subseteq C$ then $A \cap C = A$. So if both $A \subseteq B$ and $A \subseteq C$ are true, then both $A \cap B = A$ and $A \cap C = A$. In particulare, $A \cap B = A \cap C$.

Short answer [2 points]

11. Choose one of the true/false problems above and explain why it is true or false. Write your answer clearly and carefully. Neatness counts.