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16.

Part I: True/False [1 pt each]

For each of the following statements, decide whether it is true or false. Put **T** or **F** on the answer sheet.

1. $\dfrac{1}{3+4i} = \dfrac{3}{25} - \dfrac{4}{25}i$.

2. There is only one number $\alpha \in \mathbb{C}$ so that $\alpha^3 = 1$.

3. The set $\{f : [0,1] \to \mathbb{R} : f \text{ is one-to-one}\}$ is a subspace of $\mathbb{R}^{[0,1]}$.

4. The set $\{(a,b,c,d) \in \mathbb{R}^4 : c = 2a\}$ is a subspace of $\mathbb{R}^4$.

5. The list $(1+i, 1+i, 1+i), (1+i, 0, 0)$ is independent in $\mathbb{C}^3$.

6. The vectors $(1,1,1,1,1)$, $(0,1,0,1,0)$, $\left(\frac{1}{2}, 0, \frac{1}{3}, 0, \frac{1}{4}\right)$, $(1,2,3,4,5)$ span $\mathbb{R}^5$.

7. There exist real numbers a, b, c, d, e, f, not all zero, with

$$a(1,1,1,1,1) + b(0,1,0,1,0) + c\left(\frac{1}{2}, 0, \frac{1}{3}, 0, \frac{1}{4}\right) + d(1,2,3,4,5) + e(1,2,9,1,1) + f(-1,-1,-2,-2,-3) = (0,0,0,0,0).$$

8. The list of polynomials $(x-1), (x-1)^2, (x-1)^3$ is a basis for $\mathcal{P}_3(\mathbb{R})$.

9. The list of polynomials $(x-1), (x-1)^2, (x-1)^3$ is a basis for the space $\{p \in \mathcal{P}_3(\mathbb{R}) : p(1) = 0\}$.

Part II: more True/False [1pt each]

For problems 10–15, consider the following subspaces of $\mathbb{R}^3$

$$W = \{(0,0,a) \in \mathbb{R}^3 : a \in \mathbb{R}\}$$
$$X = \{(a,a,a) \in \mathbb{R}^3 : a \in \mathbb{R}\}$$
$$Y = \{(a,b,c) \in \mathbb{R}^3 : a+b+c = 0\}$$
$$Z = \{(a,a,b) \in \mathbb{R}^3 : a, b \in \mathbb{R}\}$$

10. $(1,1,-2) \in Y \cap Z$

11. $W \subseteq Z$

12. $W \cap X = \{(0,0,0)\}$

13. $\dim(Y) = 1$

14. $\mathbb{R}^3 = Y \oplus Z$

15. $Z = W \oplus X$

Part III: Short answer [2 points]

16. Choose one of the true/false problems above and explain why it is true or false. Write your answer clearly and carefully. Neatness counts.