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13.

Part I: Multiple Choice. 1 point each

1. If A is a $50,000 \times 30$ matrix and B is a $30 \times 50,000$ matrix, then the product AB is a matrix with

(a) 2.5 billion entries

(b) 3 million entries

(c) 1.5 million entries

(d) 900 entries

2. The matrix $C = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ is invertible. The $(2,1)$ entry of C^{-1} is

(a) $\frac{4}{11}$ (b) $\frac{1}{4}$ (c) 2 (d) 4 (e) -1

3. Let $T \in \mathcal{L}(\mathcal{P}_5(\mathbb{R}), \mathbb{R}^3)$. If you choose bases for $\mathcal{P}_5(\mathbb{R})$ and $\mathbb{R}^3$ and express T as a matrix $\mathcal{M}(T)$, then how many rows will the matrix $\mathcal{M}(T)$ have?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5 (f) 6

4. Suppose that $S \in \mathcal{L}(\mathbb{R}^3, \mathcal{P}_2(\mathbb{R}))$ and that $S(1,0,0) = x^2 + 2$, $S(0,1,0) = 2x^2 + 4$, and $S(0,0,1) = 3x^2 + 6$. Then the dimension of range(S) is

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

5. The map $T : \mathcal{P}_5(\mathbb{R}) \to \mathbb{R}^3$ defined by $T(p) = \left(p(0), \int_0^1 p(x)\,dx, p(1)\right)$ is surjective. Which is a basis for null(T)?

(a) $3x^5 - 5x^4 + 2x^3$

(b) $3x^5 - 5x^4 + 2x^3, \quad 5x^4 - 20x^3 + 15x^2$

(c) $3x^5 - 5x^4 + 2x^3, \quad 5x^4 - 8x^3 + 3x^2$

(d) $3x^5 - 5x^4 + 2x^3, \quad 5x^4 - 8x^3 + 3x^2, \quad 3x^5 - 2x^3 - 3x^2 + 2x$

(e) $3x^5 - 5x^4 + 2x^3, \quad 5x^4 - 8x^3 + 3x^2, \quad 3x^5 - 2x^3 - 3x^2 + 2x, \quad 4x^3 - 6x^2 + 2x$

6. Let $T : \mathcal{P}_5(\mathbb{R}) \to \mathbb{R}^3$ be the map defined by $T(p) = \left(p(0), \int_0^1 p(x)\,dx, p(1)\right)$. Using the basis $1, x, x^2, x^3, x^4, x^5$ for $\mathcal{P}_5(\mathbb{R})$ and the standard basis for $\mathbb{R}^3$, the matrix for T is

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 1 \\ 1 & 5 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 4 & 3 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & \frac{1}{2} & 1 \\ 0 & \frac{1}{3} & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

7. Suppose that $S : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear map and suppose that $S(1, 0, -5) = (0, 0, 0)$. Four of the following statements about S are true and one is false. Which one is false?

(a) S is surjective.

(b) S is not injective.

(c) S is not surjective.

(d) There are numbers $a, b, c \in \mathbb{R}$ so that the equation $S(x, y, z) = (a, b, c)$ has no solution for $(x, y, z) \in \mathbb{R}^3$.

(e) The equation $S(x, y, z) = (0, 0, 0)$ has infinitely many solutions for $(x, y, z) \in \mathbb{R}^3$.

8. Let $D : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$ be the derivative operator defined by $D(p) = p'$. One of the following statements about D is true and the rest are false. Which is true?

(a) D is surjective

(b) D is injective

(c) D is invertible

(d) D is bijective

9. Consider the derivative operator $D : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ defined by $D(p) = p'$. Using the basis

$$2x^2 + x + 1, \quad 4x + 1, \quad 4$$

for both the domain and codomain to express D as a matrix results in $\mathcal{M}(D) =$

(a) $\begin{pmatrix} 2 & 2 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

Part II: True/False. 1 point each

It will be helpful to know that

$$\begin{pmatrix} 1 & 6 & -7 \\ -2 & -5 & 4 \\ -1 & -4 & 4 \end{pmatrix}\begin{pmatrix} 4 & -4 & 11 \\ -4 & 3 & -10 \\ -3 & 2 & -7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

10. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map defined by

$$T(x, y, z) = (4x - 4y + 11z, -4x + 3y - 10z, -3x + 2y - 7z).$$

The map T is an isomorphism.

11. The system of equations

$$\begin{aligned} 4x - 4y + 11z &= \sqrt{5} \\ -4x + 3y - 10z &= \frac{13}{17} \\ -3x + 2y - 7z &= -11 \end{aligned}$$

has a unique solution for $x, y, z \in \mathbb{R}$.

12. The system of homogeneous equations

$$\begin{aligned} 4x - 4y + 11z &= 0 \\ -4x + 3y - 10z &= 0 \\ -3x + 2y - 7z &= 0 \end{aligned}$$

has no nontrivial solutions for $x, y, z \in \mathbb{R}$.

Part III: Short Answer. 3 points

13. Choose one of the previous problems and write a complete justification of your answer.