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1. The dimension of $\mathcal{P}_2(\mathbb{R})$ is

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4 (f) 5

2. True or False: There exist real numbers a, b so that $a(-7,4) + b(5,-2) = (12,11)$.

3. Which one of the following lists of vectors is indpendent?

(a) $(2,3,0,4),(4,6,0,8)$

(b) $(1,2,3),(0,0,0)$

(c) $(1,1,1),(1,0,0),(2,1,1)$

(d) $(1,1,1),(1,2,3),(-3,8,1),(3,10,15)$

(e) $(0,2,0,4),(0,2,0,5),(1,2,3,4)$

4. True or False: If $\dim(V) = 4$ and v_1, v_2, v_3, v_4 is a linearly independent list of four vectors from V then $\operatorname{span}(v_1, v_2, v_3, v_4) = V$.

5. Which one of the following lists of polynomials is linearly dependent?

(a) $1, x, x^3$

(b) $1, x, x^2, x^3, x^4, x^5$

(c) $x^2 + 2x + 3, x^2 - x, 3x^2 + x + 1, 2x^2 + 1$

(d) $1 - x, 1 + x$

(e) $5x^2 + 1, x^2 + 1, x^2 + x + 4$

6. Which of the following is a basis for $\mathbb{R}^3$?

(a) $(1,2,0),(0,0,5),(1,0,3),(1,2,3)$

(b) $(1,2,0),(0,0,5),(1,0,3)$

(c) $(1,2,0),(0,1,5)$

(d) $(1,0,0),(0,0,1),(1,0,1)$

(e) $(1,2,3),(4,5,8),(9,6,7),(3,2,8)$

7. True or False: The vector space $\{f : [0,1] \to \mathbb{R} : \ f \text{ is continuous}\}$ is infinite dimensional.

8. True or False: If $a, b, c, d, e, f, g, h, i$ are real numbers satisfying

$$a + b + c = 0 \qquad d + e + f = 0 \qquad g + h + i = 0$$

then $(a, b, c), (d, e, f), (g, h, i)$ is a list of linearly dependent vectors in $\mathbb{R}^3$.

9. Which one of the following sets of polynomials is *not* a subspace of $\mathcal{P}(\mathbb{R})$?

(a) {polynomials of degree 3}

(b) {polynomials $p(x)$ satisfying $p(1) = 0$ and $p'(1) = 0$}

(c) {even degree polynomials}

(d) {polynomials $p(x)$ with $\int_0^1 p(x)dx = 0$}

(e) {polynomials of degree ≤ 100}

10. True or False: The list of polynomials $1, (x-5)^2, (x-5)^3$ is a basis for the subspace U of $\mathcal{P}_3(\mathbb{R})$ defined by $U = \{p \in \mathcal{P}_3(\mathbb{R}) : p'(5) = 0\}$.

11. True or False: A list of vectors $v_1, \ldots, v_n$ is a basis for a vector space V if and only if every vector $v \in V$ can be expressed as a unique linear combination of the vectors $v_1, \ldots, v_n$.

12. Let

$$U = \{p(x) \in \mathcal{P}_4(\mathbb{R}) : p(2) = p(5)\} \text{ and } W = \{p(x) \in \mathcal{P}_4(\mathbb{R}) : p(2) = p(5) = p(6)\}.$$

Which of the following statements is true:

(a) U is a subspace of W

(b) $\dim(U) < \dim(W)$

(c) $U \oplus W = \mathcal{P}_4(\mathbb{R})$

(d) $\dim(W) = 3$

(e) $x^2 - 7x + 11 \in U \cap W$

13. [2 points] Choose one of the previous problems and give a complete justification of your answer. Write your answer clearly and concisely on the back of the answer sheet. Here's some guidance as to what constitutes a complete justification:

- If you choose a true/false problem that is true, give a proof.
- If you choose a true/false problem that is false, give a counterexample.
- If you choose a multiple choice problem, explain why your answer is correct and why the other choices are wrong.