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14.

Part I: Multiple Choice. 1 point each

1. Let

$$A = \begin{pmatrix} 3 & 1 & 1 & -1 & -3 \\ 0 & -2 & 1 & 0 & -3 \\ -1 & -2 & 2 & -1 & 0 \\ 3 & -1 & 1 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 0 & -2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 3 & -2 \\ -3 & 0 & -2 \end{pmatrix}.$$

The $(2,3)$ entry of matrix AB is

(a) -6

(b) -3

(c) 0

(d) 3

(e) 6

2. Using the basis $(x-1),(x-2)$ for $\mathscr{P}_1(\mathbb{R})$, the matrix $\mathscr{M}(x-5)$ is

(a) $\left(\frac{9}{2},-4\right)$

(b) $(3,2,1)$

(c) $(-3,4)$

(d) $(1,0,-1)$

(e) $(8,-3)$

3. Consider the linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x,y,z) = (x-2y-2z, 2x-6y-7z, -4x+10y+10z).$$

Using the standard basis for the domain and the standard basis for the codomain, the first row of the matrix for T is

(a) $(1,0,0)$

(b) $(1,2,-4)$

(c) $(5,4,-2)$

(d) $(1,-2,-2)$

(e) $(5,0,1)$

4. Consider the linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x,y,z) = (x-2y-2z, 2x-6y-7z, -4x+10y+10z).$$

Using the basis $(5,4,-2), (0,1,-1), \left(1, \frac{3}{2}, -1\right)$ for the domain and the standard basis for the codomain, the first row of the matrix for T is

(a) $(1,0,0)$

(b) $(1,2,-4)$

(c) $(5,4,-2)$

(d) $(1,-2,-2)$

(e) $(5,0,1)$

Part II: True/False. 1 point each

5. If $S : \mathbb{R}^3 \to \mathbb{R}$ be given by $S(x,y,z) = x+y+z$ then $\text{null}(S) = \text{span}((5,-3,-2),(0,3,-3))$.

6. The linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z) = (x+y,z,x+y-z)$ is surjective.

7. The linear map $T : \mathscr{P}_1(\mathbb{R}) \to \mathbb{R}^3$ defined by $T(p(x)) = (p(1),p(2),p(3))$ is surjective.

8. Let $T : \mathscr{P}_1(\mathbb{R}) \to \mathbb{R}^3$ be the linear map defined by

$$T(p(x)) = (p(1),p(2),p(3)).$$

Using the basis $(x-1),(x-2)$ for $\mathscr{P}_1(\mathbb{R})$ and the standard basis is used for $\mathbb{R}^3$, the matrix equation $\mathscr{M}(T)\mathscr{M}(x-5) = \mathscr{M}(T(x-5))$ is the equation

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \\ -2 \end{pmatrix}$$

9. The follow homogeneous system of equations has infinitely many solutions:

$$\begin{aligned} x - 2y - 2z &= 0 \\ 2x - 6y - 7z &= 0 \\ -4x + 10y + 10z &= 0 \end{aligned}$$

10. The follow inhomogeneous system of equations has exactly one solution:

$$\begin{aligned} x - 2y - 2z &= \frac{1}{5} \\ 2x - 6y - 7z &= -17 \\ -4x + 10y + 10z &= 9 \end{aligned}$$

11. There is a linear map $T : \mathbb{R}^5 \to \mathbb{R}^2$ with

$$\text{null}(T) = \{(x_1,x_2,x_3,x_4,x_5) \in \mathbb{R}^5 : x_1 = 3x_2 \text{ and } x_3 = x_4 = x_5\}.$$

12. Suppose $T : V \to W$. If $v_1,\ldots,v_n$ is independent in V, then $Tv_1,\ldots,Tv_n$ is independent in W.

13. Suppose $T : V \to W$. If $v_1,\ldots,v_n$ spans V, then $Tv_1,\ldots,Tv_n$ spans the range(T).

Part III: Short Answer. 3 points

14. Choose one of the True / False problems and write a complete justification of your answer.