Part I: Multiple Choice [1 point each]

1. One of the following lists is linearly independent. Which one?

(a) $(1, 2, 3),\ (4, 5, 6),\ (7, 8, 9),\ (10, 11, 12)$

(b) $1 + x + x^2 + 4x^3,\ 2 + 2x + 2x^2 + 8x^3$

(c) $(1, 2, 3, 4),\ (0, 0, 0, 1),\ (1, 2, 3, 5)$

(d) $1 + x^6,\ 1 - x^5,\ 1 + x^2 - x^3,\ 1 + x$

(e) $1 + x + x^2,\ 4 - x^2,\ x + x^2,\ 6 - x$

2. Let $T : \mathcal{P}_4(\mathbb{R}) \to \mathbb{R}^4$ be the linear map defined by $T(p) = (p(0), p(1), p(2), p(3))$. Which is a basis for null(T)?

(a) $x(x - 1)(x - 2)(x - 3)$

(b) $2x^4 - 12x^3 + 22x^2 - 12x,\ 2x^5 - 12x^4 + 22x^3 - 12x^2$

(c) $x^3 - 6x^2 + 11x - 6$

(d) $x^3 - 3x^2 + 2x, x^3 - 4x^2 + 3x,\ x^3 - 6x^2 + 11x - 6$

(e) $x^3 - 3x^2 + 2x,\ x^3 - 4x^2 + 3x$

3. Which statement about the matrix $A = \begin{pmatrix} 3 & -1 & 5 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 2 & -3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$ is true?

(a) There exists a basis of $\mathbb{R}^4$ consisting of eigenvectors for A.

(b) The nullspace of A is trivial.

(c) The matrix A is invertible.

(d) -1 is an eigenvalue for A

(e) $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ is an eigenvector for A

4. Which of the following vectors are *not* eigenvectors for the matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

5. Which statement about the operator $D : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$ defined by $D(p) = p'$ is false?

(a) D is surjective

(b) 0 is an eigenvalue for D

(c) D is injective

(d) $\mathcal{P}_2(\mathbb{R})$ is an invariant subspace for D

(e) $\text{null}(D) \subseteq \text{range}(D)$

Part II: True or False [1 point each]

6. The matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ is invertible.

7. The map $T : \mathcal{P}_3(\mathbb{R}) \to \mathbb{R}^4$ defined by $T(p) = (p(0), p(1), p(2), p(3))$ is invertible.

8. For any numbers a, b, c, the system of equations $y + z = a$, $x + z = b$, $x + y = c$ has a unique solution $(x, y, z) \in \mathbb{R}^3$.

9. If $T : V \to V$ is a linear operator on a vector space V then $V = \text{null}(T) \oplus \text{range}(T)$.

10. If a matrix A satisfies $A^2 = I$ then $A = I$ or $A = -I$.

Part III: Short Answer [2 points]

11. Choose one of the true/false problems above and explain your answer. Write your explanation clearly and concisely.

Part IV: Compute [2 points]. Show your work.

12. Find the eigenvalues of the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$.

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11.

12.