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15.

Multiple Choice: 1 point each

1. Let $T : \mathscr{P}_2(\mathbb{R}) \to \mathscr{P}_2(\mathbb{R})$ be defined by $T(p) = p(1)x^2 + p'(0)x + p''(0)$. Let $\mathscr{M}(T)$ be the matrix for T using the basis $1+x^2, 2-x^2, 2-2x+x^2$. The $(2,2)$ entry of $\mathscr{M}(T)$ is

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

2. Which of the following statements about the linear operator $T : \mathscr{P}_2(\mathbb{R}) \to \mathscr{P}_2(\mathbb{R})$ defined by $T(p) = p(1)x^2 + p'(0)x + p''(0)$ is false?

(a) There is a basis of $\mathscr{P}_2(\mathbb{R})$ consisting of eigenvectors for T.

(b) T is invertible

(c) The nullspace of T is one-dimensional.

(d) T has three distinct eigenvalues.

(e) $1+x^2$ is an eigenvector for T with eigenvalue 2

3. For what two values of x is $\begin{pmatrix} 5 \\ x \end{pmatrix}$ an eigenvector for $\begin{pmatrix} -14 & 5 \\ -24 & 9 \end{pmatrix}$?

(a) 1 and 6 (b) -3 and 4 (c) 5 and -2 (d) 7 and -2 (e) 8 and 15

4. The $(1,1)$ entry of $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}^{-1}$ is

(a) -3 (b) $-\frac{5}{2}$ (c) -2 (d) -1 (e) 3

5. Let V be a n-dimensional vector space and let $T \in \mathscr{L}(V)$. Which of the following statements is not equivalent to the others?

(a) V has a basis of eigenvectors for T.

(b) There is a basis for V in which the matrix for T is diagonal.

(c) T has n distinct eigenvalues.

6. Let V be a n-dimensional vector space and let $T \in \mathscr{L}(V)$. Which of the following statements is not equivalent to the others?

(a) $\text{null}(T - \lambda \,\text{Id}) = \{0\}$.

(b) λ is an eigenvalue for T.

(c) $T - \lambda \,\text{Id}$ is not injective.

(d) $T - \lambda \,\text{Id}$ is not surjective.

(e) $T - \lambda \,\text{Id}$ is not bijective.

(f) $T - \lambda \,\text{Id}$ is not invertible.

7. Let V be an n-dimensional vector space and let $T \in \mathscr{L}(V)$. Which of the following statements is not equivalent to the others?

(a) $\text{null}(T) = \{0\}$.

(b) T is invertible.

(c) $\text{range}(T) = V$.

(d) For any $y \in V$ there exists a unique $x \in V$ with $Tx = y$.

(e) $Tv_1, \ldots, Tv_n$ is a basis for V whenever $v_1, \ldots, v_n$ is a basis of V.

(f) V has a basis of eigenvectors for T.

8. Consider the backward shift map $S : \mathbb{R}^\infty \to \mathbb{R}^\infty$ defined by $S(a_1, a_2, a_3, \ldots) = (a_2, a_3, \ldots)$. Which of the following statements about S is false?

(a) S is surjective.

(b) S is injective.

(c) 0 is an eigenvalue for S.

(d) 1 is an eigenvalue for S.

(e) $(1, 1, 1, \ldots)$ is an eigenvector for S.

True or False: 1 point each

9. There exists a linear map $T : \mathbb{R}^5 \to \mathbb{R}^3$ with $\text{null}(T) = \{(x, 2x, 3x, 4x, 5x) : x \in \mathbb{R}\}$.

10. If $S, T \in \mathscr{L}(V)$ and $ST = TS$ then $\text{null}(S)$ is an invariant subspace for T.

11. $\mathbb{R}^{\mathbb{R}} = U \oplus W$ where $U = \{f \in \mathbb{R}^{\mathbb{R}} : f(x) = f(-x)\}$ and $W = \{f \in \mathbb{R}^{\mathbb{R}} : f(x) = -f(-x)\}$.

12. $x, 3x^2 - 1, x^3$ is a basis for the nullspace of $I : \mathscr{P}_3(\mathbb{R}) \to \mathbb{R}$ defined by $I(p) = \int_{-1}^{1} p(x)dx$.

13. $(x-2)^2$ divides $4 - 8x + 5x^2 + 7x^3 - 4x^4 - 2x^5 + x^6$.

14. $\text{span}((1,2,-2),(2,3,-5))$ is an invariant subspace of $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z) = (12x + 5z, 7x + 3y + 5z, -33x + y - 13z)$.

Part III: Short Answer. 2 points

15. Choose one of the true false problems and write a complete justification of your answer.