1. Prove: If some vector in a list of vectors in a vector space V is a linear combination of the other vectors, then the list is linearly dependent.

2. Does $(1, 2, 3, -5), (4, 5, 8, 3), (9, 6, 7, -1)$ span $\mathbb{R}^4$? Explain.

3. Is the list $(1, 2, 3), (4, 5, 8), (9, 6, 7), (-3, 2, 8)$ linearly independent in $\mathbb{R}^3$? Explain.

4. Prove that F^∞ is infinite-dimensional.

5. Suppose that p_1, p_2, p_3, p_4, p_5 is a list polynomials in $\mathcal{P}_4(\mathbb{R})$ that all vanish at $x = 3$. Prove that p_1, p_2, p_3, p_4, p_5 is linearly dependent.

6. Suppose that v_1, v_2, v_3 is a basis for a vector space V. Prove or disprove v_1+v_2, v_1-v_2, v_3 is also a basis for V.

7. Let $U = \{(a, b, c) \in \mathbb{R}^3 : a + b + c = 0\}$.

(a) Find a basis for U.

(b) Extend your basis to a basis of $\mathbb{R}^3$.

(c) Find a subspace W of $\mathbb{R}^3$ so that $\mathbb{R}^3 = U \oplus W$.

8. Let $U = \{p \in \mathcal{P}_4(\mathbb{R}) : p(2) = p(5)\}$.

(a) Find a basis for U.

(b) Extend your basis to a basis of $\mathcal{P}_4(\mathbb{R})$

(c) Find a subspace W of $\mathcal{P}_4(\mathbb{R})$ so that $\mathcal{P}_4(\mathbb{R}) = U \oplus W$.

9. Let $U = \{p \in \mathcal{P}_4(\mathbb{R}) : \int_{-1}^{1} p = 0\}$.

(a) Find a basis for U.

(b) Extend your basis to a basis of $\mathcal{P}_4(\mathbb{R})$

(c) Find a subspace W of $\mathcal{P}_4(\mathbb{R})$ so that $\mathcal{P}_4(\mathbb{R}) = U \oplus W$.

10. Prove that any two three dimensional subspaces of $\mathbb{R}^5$ must have a nonzero vector in their intersection.