Name:

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15.

True/False [1 pt each] For problems 1–14, decide whether each statement is true or false. Put **T** or **F** on the answer sheet.

Part I

Use the functions e, f and g as defined below for problems 1–6

$$\{1,2,3\} \xrightarrow{e} \{1,2,3\} \qquad \{1,2,3\} \xrightarrow{f} \{1,2,3\} \qquad \{1,2,3\} \xrightarrow{g} \{1,2,3\}$$

$$1 \longmapsto 1 \qquad 1 \longmapsto 2 \qquad 1 \longmapsto 3$$

$$2 \longmapsto 2 \qquad 2 \longmapsto 1 \qquad 2 \longmapsto 1$$

$$3 \longmapsto 3 \qquad 3 \longmapsto 3 \qquad 3 \longmapsto 2$$

1. $2 \xmapsto{fg} 2$.

2. $fg = e$.

3. f is bijective.

4. $gf = fg$.

5. $ggf = fg$.

6. ggf is an inverse for fg.

Part II: More True/False

7. The relation $\sim$ defined on $\mathbb{Z}$ by $m \sim n \Leftrightarrow \gcd(m, n) = 1$ is an equivalence relation.

8. The composition of two monomorphisms is a monomorphism.

9. A function $f : X \to Y$ between two sets X and Y is surjective if and only if there exists a function $h : Y \to X$ with $fh = \mathrm{id}_Y$.

10. Let X and Y be sets and suppose $f : X \to Y$. If for all functions $g : Y \to Z$ and $h : Y \to Z$, $gf = hf \Rightarrow g = h$, then f is surjective.

11. Let X and Y be sets and consider functions $f : X \to Y$, $g : Y \to X$, and $h : Y \to X$. If $hf = \mathrm{id}_X$ and $fg = \mathrm{id}_Y$ then $g = h$.

Part III

Use the following information for the problems 12, 13, and 14. Let $\mathcal{A}, \mathcal{B}$, and $\mathcal{C}$ be the following sets

$\mathcal{A} = \{\text{A,B,C}, \ldots, \text{Y, Z}\}$

$\mathcal{B} = \{$Isabella, Madison, Olivia, Liam, Joseph, Jayden, Esther, Elijah, Moshe, Emma, Mia$\}$

$\mathcal{C} = \{\text{yellow}, \text{purple}\}$.

and let $f : \mathcal{B} \to \mathcal{A}$ and $g : \mathcal{B} \to \mathcal{C}$ be the functions defined by

$$f(x) = \text{the first letter of the name } x$$

$$g(x) = \begin{cases} \text{yellow} & \text{if name } x \text{ has an even number of letters} \\ \text{purple} & \text{if name } x \text{ has an odd number of letters} \end{cases}$$

12. The function defined by

$$\mathcal{C} \xrightarrow{\ h\ } \mathcal{B}$$

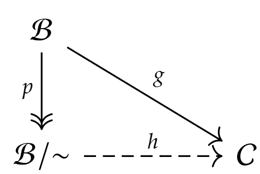
$$\text{yellow} \longmapsto \text{Elijah}$$

$$\text{purple} \longmapsto \text{Madison}$$

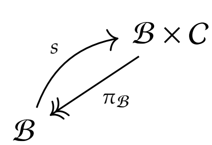
is a right inverse for g

13. Define an equivalence relation $\sim$ on $\mathcal{B}$ by $x \sim x' \Leftrightarrow f(x) = f(x')$. That is, two names are equivalent if and only if they begin with the same letter. There are Let $p : \mathcal{B} \twoheadrightarrow \mathcal{B}/\sim$ be the natural surjection mapping a name to its equivalence class.

The map g factors through p. That is, there exists a well defined map $h : \mathcal{B}/\sim \to \mathcal{C}$ so that $hp = g$ as pictured below:



14. Consider the natural projection $\pi_{\mathcal{B}} : \mathcal{B} \times \mathcal{C} \to \mathcal{B}$. Define a map $s : \mathcal{B} \to \mathcal{B} \times \mathcal{C}$ by $s(x) = (x, g(x))$. Then s is a section of $\pi_{\mathcal{B}}$. That is $\pi_{\mathcal{B}} s = \text{id}_{\mathcal{B}}$. Here is a picture:



Part IV

Short answer [3 points]

15. Choose one of the True/False problems above and explain why it is true or false. Write your answer clearly and carefully. Neatness counts.