Name: ______________________________

1	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ & & & & \end{pmatrix}$
2	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ & & & & \end{pmatrix}$
3	$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ & & & & \end{pmatrix}$
4	
5	

6	
7	
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11	
12	
13	
14	

Name:

15.

Short Answer [1 pt each] Problems 1 through 5 concern computations in the symmetric group S_5. For these problems, $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 5 & 3 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 1 & 4 & 3 \end{pmatrix}$.

1. $\sigma\tau =$

2. $\sigma^{-1} =$

3. If $\sigma^2 x = \tau$ then $x =$

4. $|\sigma| =$

5. $|S_5| =$

True/False [1 pt each] For problems 6–14, decide whether each statement is true or false. Put **T** or **F** on the answer sheet.

6. The binary operation $\star$ defined on $\mathbb{Z}$ by $a \star b = 3ab$ is associative.

7. Suppose G is a group and $x, y \in G$ satisfy $x^2 = y^3 = e$ and $yx = xy^2$. Then $(yx)^{-1} = xy^2$.

8. If G is a group and $g^2 = e$ for every $g \in G$, then G is abelian.

9. Every element of S_3 has order 1, 2, or 3.

10. $|\,(\mathbb{Z}/12\mathbb{Z})^*\,| = 5$.

11. $|[3]_7| = 6$ in $(\mathbb{Z}/7\mathbb{Z})^*$.

12. If $\phi : G \to H$ is a group homomorphism, then $|\phi(g)| = |g|$ for all $g \in G$.

13. S_3 and $\mathbb{Z}/6\mathbb{Z}$ are isomorphic.

14. $\mathbb{Z}/6\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$ are isomorphic.

Short answer [6 points]

15. Choose one of the True/False problems above and explain why it is true or false. Write your answer clearly and carefully on the back of your answer sheet. Neatness counts.