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**Name:**

# Choose one. 10, 11, or 12:

## True or False [1 pt each]

**1.** In a group $G$ if $ab = ac$ then $b = c$.

**2.** For functions $f : B \to C$, $g : A \to B$ and $h : A \to B$, if $fg = fh$ then $g = h$.

**3.** If $A \neq \emptyset$ and $f : A \to B$ is an injective function, then there exists a function $g : B \to A$ with $gf = \mathrm{id}_A$.

**4.** If $\phi : G \to G'$ is an injective group homomorphism, then there exists a group homomorphism $\psi : G' \to G$ with $\psi\phi = \mathrm{id}_G$.

**5.** The function $f : \mathbb{Z} \to GL_2(\mathbb{R})$ defined by $f(a) = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ is a group homomorphism.

**6.** The subgroup $S_3 \subset S_4$ is a normal subgroup of $S_4$.

**7.** If $H$ and $K$ are subgroups of a group $G$ and $|H| = 8$ and $|K| = 15$, then $H \cap K = \{e\}$.

**8.** The groups $(\mathbb{Z}/12\mathbb{Z})^*$ and $(\mathbb{Z}/5\mathbb{Z})^*$ are isomorphic.

**9.** If $\phi : G \to G'$ is a group homomorphism then $\phi(a) = \phi(b)$ if and only if $a$ and $b$ are in the same coset of $\ker(\phi)$.

## Mathematical writing [3 pts]

Choose *one* of the following problems.

**10.** The subgroup $K = \{1, (12)(34), (13)(24), (14)(23)\}$ is a normal subgroup of $S_4$. Your problem: compute the cosets of $K$ and compute a multiplication table for $S_4/K$.

**11.** Define what it means for a subgroup $N$ of a group $G$ to be *normal*. Give an example of a group $G$ with a subgroup $N$ that is normal and a subgroup $H$ that is not normal. Justify your answer.

**12.** Let $\phi : G \to G'$ be a group homomorphism. Define what it means for $\phi$ to be *monic* and for $\phi$ to be *left-invertible*. Prove that if $\phi$ is left invertible then $\phi$ is monic, but not conversely.