1. Prove that $SL_2(\mathbb{R})$ is a normal subgroup of $GL_2(\mathbb{R})$.

2. Let $G = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/8\mathbb{Z}$ and $H = \{(0,0),(1,2),(0,4),(1,6)\}$. Compute the cosets of H and write down a multiplication table for G/H.

3. The dihedral group D_8 has a presentation as a finitely generated group as

$$D_8 = \langle x, y : x^2 = y^4 = 1, \quad yx = xy^3 \rangle.$$

One can identify x and y with the following symmetries of a square: x is reflection through the horizontal line passing through the middle of the square and y denotes the counterclockwise rotation by 90 degrees. Make a multiplication table for D_8 using $\{1, y, y^2, y^3, x, xy, xy^2, xy^3\}$.

4. The subgroup $K = \{1, y^2\}$ is a normal subgroup of D_8. Compute the cosets of K and compute a multiplication table for D_4/K.

5. Let G be a group. An element $b \in G$ is a conjugate of $a \in G$ if there exists $g \in G$ so that $b = g^{-1}ag$. Define a relation $\sim$ on G by

$$b \sim a \Leftrightarrow \ b \text{ is a conjugate of } a.$$

(a) Prove that $\sim$ defines an equivalence relation on G.

(b) For $G = S_3$, describe the equivalence classes of $\sim$. Are these equivalence classes the cosets of a subgroup of S_3?

(c) Prove or disprove: in every group, ba is conjugate to ab.

(d) The matrices $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ are conjugate in $GL_2(\mathbb{R})$.

6. Compute the order of $(8\ 5\ 1\ 7)(4\ 5\ 6\ 9)(10\ 2\ 3\ 4\ 1) \in S_{10}$.

7. Prove that if H and K be subgroups of a group G then $H \cap K$ is a subgroup of G.

Prove or disprove:

8. In a group, if a and b have finite, then ab has finite order.

9. In a group, if a and b have finite order, then ab and ba have the same order.

10. In a group, if $|a| = n$ and $|b| = k$, then $|ab| = \mathrm{lcm}(n,k)$.

11. $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ and $\mathbb{Z}/4\mathbb{Z}$ are isomorphic.

12. $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$ and $\mathbb{Z}/6\mathbb{Z}$ are isomorphic.

13. There exists a nontrivial homomorphism $S_3 \to \mathbb{Z}/5\mathbb{Z}$.

14. There exists a nontrivial homomorphism $S_3 \to \mathbb{Z}/3\mathbb{Z}$.

15. There exists a nontrivial homomorphism from S_3 to a group with 81 elements.

16. The map $\phi : \mathbb{Z}/2\mathbb{Z} \to S_3$ defined by $\phi(0) = e$ and $\phi(1) = (12)$ is a monic homomorphism.

17. The map $\phi : \mathbb{Z}/2\mathbb{Z} \to S_3$ defined by $\phi(0) = e$ and $\phi(1) = (12)$ is left invertible homomorphism.

18. Let G and G' be groups with $|G| = 18$ and $|G'| = 15$. There exists a homomorphism $\phi : G \to G'$ with $|\ker\phi| = 6$.