1. [Problem 6 from Spring 2021]

Suppose that the following is a pullback diagram in TOP:

$$
\begin{array}{ccc}
F & \longrightarrow & E \\
{\scriptstyle P}\downarrow & \lrcorner & \downarrow{\scriptstyle p} \\
C & \longrightarrow & B
\end{array}
$$

Prove that if p is monic then P is monic.

2. [Problem 3 from Fall 2022]

Let X and Y be spaces and let $f : X \to Y$ be a function. Prove that f is a continuous injection if and only if the following diagram is a pullback square:

$$
\begin{array}{ccc}
X & \xrightarrow{\ id\ } & X \\
{\scriptstyle id}\downarrow & & \downarrow{\scriptstyle f} \\
X & \xrightarrow[\ f\ ]{} & Y
\end{array}
$$

3. [Problem 3 from Fall 2016]

Prove that the following two descriptions of the torus are homeomorphic:

1. $S^1 \times S^1$

2. A square with opposite sides identified as shown.
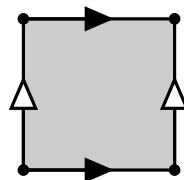
4. [Problem 2 from Spring 2016]

Suppose X is contractible and Y is any space. Prove or disprove $X \times Y$ is homotopy equivalent to Y.

5. [Problem 4 from Spring 2012 ≈ problem 4 from December 2020]

Suppose A is a subspace of X and $f : A \to Y$. We say f *can be extended to* X if and only if there exists a map $g : X \to Y$ with $g = f$ on A.

(a) Prove that if A is a dense subspace of X and Y is Hausdorff, then $f : A \to Y$ can be extended to X in at most one way.

(b) Give an example of spaces X, Y, a dense subset $A \subseteq X$, and a map $f : A \to X$ that cannot be extended to X.

(c) Give an example of spaces X, Y, a dense subset $A \subseteq X$, and a map $f : A \to X$ that can be extended to X in more than one way.

6. [Problem 1 from Fall 2021 and Problem 2 on Spring 2021]

Prove or disprove:

(a) **[Problem 1a from Fall 2021]** Compact is a homotopy invariant.

(b) **[Problem 1b from Fall 2021]** Connected is a homotopy invariant.

(c) **[Problem 2 on Spring 2021]** Hausdorff is a homotopy invariant.