Solutions

1	a
2	e
3	d
4	a
5	d
6	d
7	b
8	b
9	d
10	e

1. Which one of the following statements is a proposition?

(a) $5 + 7 = 10$

(b) $x + 2 = 11$

(c) Answer this question.

(d) This sentence is false.

(e) Who won the baseball game?

Answer. (a) is a proposition.

2. The contrapositive of *"If you get an A on the final exam, then you'll get an A for the course"* is

(a) *If you got an A for the course, then you got an A on the final exam.*

(b) *If you get an A on the final exam, then you won't get an A for the course.*

(c) *If you don't get an A on the final exam, then you won't get an A for the course.*

(d) *If you don't get an A on the final exam, then you'll get an A for the course.*

(e) *If you don't get an A for the course, then you didn't get an A on the final exam.*

Answer. (e). The given implication is $p \rightarrow q$ where p is *"you get an A on the final exam"* and q is *"get an A for the course"*. The contrapositive is $\neg q \rightarrow \neg p$. That's (e). The others are: (a) is the converse $q \rightarrow p$ (b) is $p \rightarrow \neq q$ (c) is $\neg p \rightarrow \neg q$ and (d) is $\neg p \rightarrow q$.

3. How many rows will a truth table for the compound proposition $(p \vee q) \leftrightarrow (p \wedge s \wedge q)$ have?

(a) 3   (b) 5   (c) 6   (d) 8   (e) 32

Answer. (d).

4. Let p and q be the propositions

$$p : \textit{It is below freezing.}$$
$$q : \textit{It is snowing.}$$

Which statement is the proposition $\neg q \rightarrow \neg p$?

(a) *If it is not snowing, then it is not below freezing.*

(b) *It is not snowing and it is not below freezing.*

(c) *It is not snowing and it is below freezing.*

(d) *It is snowing or it is below freezing.*

(e) *It is not snowing and it is below freezing.*

Answer. (a) is correct. For the others: (b) is $\neg q \wedge \neg p$, (c) is $\neq q \wedge p$, (d) is $q \vee p$, (e) is $\neg q \wedge p$.

5. Again, let p and q be the propositions

$$p : \textit{It is below freezing.}$$
$$q : \textit{It is snowing.}$$

Which statement is not equivalent to the proposition $q \rightarrow p$?

(a) *If it is not below freezing, then it is not snowing.*

(b) *If it is snowing, then it is below freezing.*

(c) *It is either not snowing or it is below freezing.*

(d) *If it is below freezing, then it is snowing.*

(e) *It is necessary that it be below freezing in order for it to be snowing.*

Answer. (d) is correct. (d) is the proposition $p \rightarrow q$ which is not equivalent to $q \rightarrow p$. For the others: (a) is $\neg p \rightarrow \neq q$, (b) is $q \rightarrow p$, (c) is $\neq q \vee p$, and (e) is also $q \rightarrow p$.

6. Consider the following propositional functions

$$p(x) : x \text{ has feathers}$$
$$q(x) : x \text{ can fly}$$
$$r(x) : x \text{ lays eggs}$$
$$s(x) : x \text{ is a bird}$$

Which is the statement *"All birds have feathers and lay eggs but not all birds can fly."*

(a) $\exists x (s(x) \wedge p(x) \wedge r(x) \wedge \neg q(x))$

(b) $\forall x (\neg q(x) \rightarrow s(x)) \vee (p(x) \vee q(x))$

(c) $\exists x (\neg q(x) \rightarrow s(x)) \vee (p(x) \vee q(x))$

(d) $\forall x (s(x) \rightarrow (p(x) \wedge r(x)) \wedge \exists x (s(x) \wedge \neg q(x))$

(e) $\forall x (s(x) \rightarrow ((p(x) \wedge r(x)) \vee \neg q(x)))$

Answer. (d) is correct.

7. Which of the following propositions is true?

(a) $\forall n \in \mathbb{R} \ (n^2 \geq n)$

(b) $\forall n \in \mathbb{Z} \ (n^2 \geq n)$

(c) $\exists n \in \mathbb{Z} \ (n^2 < n)$

(d) $\exists n \in \mathbb{R} \ (n^2 < 0)$

Answer. (b) is true. The others are false: (a) is false, for example $n = 0.5 \in \mathbb{R}$ satisfies $n^2 < n$. (c) there are no integers n whose square n^2 is smaller than n. (d) there are no real numbers whose square is negative.

8. Which of the following propositions is false?

(a) $\exists n \in \mathbb{R} \ (n^2 < n)$

(b) $\forall n \in \mathbb{Z} \ (n^2 = 1 \ \rightarrow \ n = 1)$

(c) $\forall n \in \mathbb{N} \ (n^2 = 1 \ \rightarrow \ n = 1)$

(d) $\forall n \in \mathbb{Z} \ (n^2 = n \ \rightarrow \ (n = 1 \lor n = 0))$

(e) $\exists n \in \mathbb{R} \ (n^2 = n)$

Answer. (b) is false. There exists a natural number, specifically $n = -1$, with $n^2 = 1$ and $n \neq 1$. The others are true: (a) is true, for example $n = 0.5 \in \mathbb{R}$ satisfies $n^2 < n$. (c) is true, the only natural number whose square is 1 is 1 itself. (d) is true if $n^2 = n$ then $n = 1$ or $n = 0$. (e) is true, $n = 0$ or $n = 1$ works as an example.

9. Which one of the following propositions is not satisfiable?

(a) $\neg p \rightarrow p$

(b) $(p \rightarrow q) \land (p \rightarrow \neg q)$

(c) $(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p)$

(d) $(p \land \neg q) \land (\neg p \lor q)$

Answer. (d) is not satisfiable, as a truth table will reveal, or by noticing that $(\neg p \lor q) = \neg(p \land q)$.. So, the proposition in question is $(p \land q) \land \neg(p \land q)$.. For the others: (a) is satisfiable when $p = F$. (b) is satisfiable when $p = F$. (c) is satisfiable when p, q, r are all true or when p, q, r are all false.

10. Which one of the following propositions is not a tautology?

(a) $p \lor \neg p$

(b) $(p \land q) \rightarrow p$

(c) $\neg(p \land q) \leftrightarrow (\neg p \lor \neg q)$

(d) $(p \rightarrow q) \leftrightarrow (\neg p \lor q)$

(e) $(p \lor q) \rightarrow q$

Answer. (e) is not a tautology, it's false when $p = T$ and $q = F$.