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Part 1: Multiple choice. One point each.

1. Which one of the following equals $\dfrac{10+5i}{1+2i}$?

(a) 2 (b) $3i$ (c) $3+4i$ (d) $4-3i$ (e) $\frac{1}{3}+\frac{4}{3}i$

2. Which one of the following is *not* a field?

(a) The numbers $\{0,1\}$ with $+$ and $\times$ defined "mod 2"

(b) The integers $\mathbb{Z}$

(c) The rational numbers $\mathbb{Q}$

(d) The real numbers $\mathbb{R}$

(e) The complex numbers $\mathbb{C}$

3. Which one of the following statements is false?

(a) The set $A = \{(a,b,c) \in \mathbb{R}^3 : a+b=c\}$ is a subspace of $\mathbb{R}^3$.

(b) The set $B = \{p \in \mathcal{P}(\mathbb{R}) : p''(1) = 0\}$ is a subspace of $\mathcal{P}(\mathbb{R})$.

(c) The set $C = \{f \in \mathbb{R}^{\mathbb{R}} : f(x) = f(x+1) \text{ for all } x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^{\mathbb{R}}$.

(d) The set $D = \{(a,b) \in \mathbb{R}^2 : ab = 0\}$ is a subspace of $\mathbb{R}^2$.

(e) The set $E = \{f \in \mathbb{R}^{[0,1]} : \int_0^1 f(x)\,dx = 0\}$ is a subspace of $\mathbb{R}^{[0,1]}$.

4. Which one of the following lists of vectors is a basis for $\mathbb{C}^2$?

(a) $(1,i), (i,-1)$

(b) $(1,i), (i,0)$

(c) $(1,i), (i,0), (0,1)$

(d) $(1,1)$

(e) $(1,0)$

5. Let U, V, and W be the following subspace of $\mathbb{R}^3$:

$$U = \{(a,b,c) \in \mathbb{R}^3 : a = b\}$$
$$V = \{(a,b,c) \in \mathbb{R}^3 : a = 0 \text{ and } b = 0\}$$
$$W = \{(a,b,c) \in \mathbb{R}^3 : a+b+c = 0\}$$

Which one of the following statements is true?

(a) $U + V = W$

(b) $U + V = \mathbb{R}^3$

(c) $U \oplus V = \mathbb{R}^3$

(d) $U \oplus W = \mathbb{R}^3$

(e) $V \oplus W = \mathbb{R}^3$

Part II: True or False. One point each.

6. Every polynomial of degree three can be expressed as a linear combination of the polynomials $1, 2+4x, 11x^2+2x+3, 7x^3+5x^2-1$.

7. If p_1, p_2, p_3, p_4 is a list polynomials in $\mathcal{P}_3(\mathbb{R})$ that satisfy $\int_0^1 p_i(x)\,dx = 0$ then p_1, p_2, p_3, p_4 is dependent.

8. The space $\{p \in \mathcal{P}_3(\mathbb{R}) : p(0) = p(1)\}$ is three dimensional.

9. If U and W are subspaces of a vector space V, then $U \cap W$ is a subspace of V.

10. For vector spaces U, V, and W; if $U \oplus V = U \oplus W$ then $V = W$.

Part III: Short answer. Two points.

11. Choose one of the true/false problems and explain your answer. Neatness counts.