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Part 1: Multiple choice. One point each.

1. Suppose $T : \mathscr{P}(\mathbb{R}) \to \mathbb{R}^3$ is a linear map and that $T(x^2) = (10,5,2)$, $T(x^4) = (0,0,2)$, and $T(1+x) = (-5,1,1)$. Then $T(3+3x+2x^2-x^4) =$

(a) $(-15,3,3)$ (b) $(20,10,4)$ (c) $(0,0,-2)$ (d) $(5,13,5)$ (e) $(2,5,0,3)$

2. Let $S : \mathbb{R}^2 \to \mathbb{R}^4$ be defined by $S(x,y) = (x+y, 5y, x-y, 2x+y)$. Which is a basis for the range of S?

(a) $(2,5,0,3)$

(b) $(1,0,0,0),(0,1,0,0)$

(c) $(1,0,1,2),(1,5,-1,1)$

(d) $(2,5,0,3),(1,5,-1,0),(0,-5,2,1)$

(e) $(1,0,0,0),(0,1,0,0),(0,0,1,0)$

3. Let $T : \mathscr{P}_4(\mathbb{R}) \to \mathbb{R}$ be defined by $T(p) = p''(0)$. Which of the following is a basis for null(T)?

(a) $1,x,x^2$

(b) $1+x, 1+x^2, 1+x+x^2$

(c) $1,x,x^3,x^4$

(d) x^4,x^3,x^2

(e) x,x^2,x^3

4. Suppose that $S \in \mathscr{L}(\mathbb{R}^2,\mathbb{R}^4)$ satisfies range$(S) = \{(a,b,c,d) : a = c \text{ and } b = d\}$. Then, which of one of the following statements is false.

(a) S is injective.

(b) There exists a map $T \in \mathscr{L}(\mathbb{R}^4,\mathbb{R}^2)$ with $TS = I_{\mathbb{R}^2}$.

(c) $\dim(\text{range}(S)) = 2$.

(d) $\dim(\text{null}(S)) = 2$.

(e) $\dim(\text{range}(S)) + \dim(\text{null}(S)) = 2$.

5. Consider the map $T : \mathscr{P}_3(\mathbb{R}) \to \mathbb{R}^4$ defined by $T(p) = (p(0), p(1), p'(0), p'(1))$ Which is the matrix for T using the basis $1,x,x^2,x^3$ for the domain and the standard basis for the codomain?

(a) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 3 \end{pmatrix}$

Part II: True or False. One point each.

6. Let $T \in \mathscr{L}(V,W)$ be a linear map between finite dimensional vector space. For every subspace Y of W, we have $\dim(T^{-1}(Y)) \geq \dim Y$.

7. The map $T : \mathscr{P}_3(\mathbb{R}) \to \mathbb{R}^4$ defined by $T(p) = (p(0), p(1), p'(0), p'(1))$ is an isomorphism.

8. There exists a nonzero polynomial $p \in \mathscr{P}_3(\mathbb{R})$ so that $p(0) = 0$, $\int_0^2 e^x p(x) = 0$, and $p'(1) = 0$.

9. For any map $T \in \mathscr{L}(V)$, if $T^2 = 0$ then $\operatorname{range}(T) \subseteq \operatorname{null}(T)$.

10. There exists a map $T \in \mathscr{L}(\mathbb{R}^2)$ with $\operatorname{null}(T) = \operatorname{range}(T)$.

11. There exists a map $T \in \mathscr{L}(\mathbb{R}^3)$ with $\operatorname{null}(T) = \operatorname{range}(T)$.

Part III: Short answer. Two points. Neatness counts.

12. Choose one of the true problems that is true and prove it.

13. Choose one of the true problems that is false and explain why it is false.