1. Which of the following is *not* a field? Explain.

(a) The integers $\mathbb{Z}$

(b) The rational numbers $\mathbb{Q}$

(c) The real numbers $\mathbb{R}$

(d) The complex numbers $\mathbb{C}$

2. Which of the following is *not* a field? Explain.

(a) The numbers $\{0, 1\}$ with $+$ and $\times$ defined "mod 2".

(b) The numbers $\{0, 1, 2\}$ with $+$ and $\times$ defined "mod 3".

(c) The numbers $\{0, 1, 2, 3\}$ with $+$ and $\times$ defined "mod 4".

(d) The numbers $\{0, 1, 2, 3, 4\}$ with $+$ and $\times$ defined "mod 5".

3. Let $\alpha \in \mathbb{C}$ be nonzero. Define the number $\frac{1}{\alpha}$ and prove that $\frac{1}{\frac{1}{\alpha}} = \alpha$.

4. Express $\dfrac{1}{4+5i}$ in the form $a + bi$ for real numbers a, b.

5. True or False:

(a) There exists a number $\alpha \in \mathbb{R}$ so that $\alpha^2 = -2$.

(b) There exits a number $\alpha \in \mathbb{C}$ so that $\alpha^2 = -2$.

6. True or False:

(a) There is only one number $\alpha \in \mathbb{R}$ so that $\alpha^3 = 2$.

(b) There is only one number $\alpha \in \mathbb{C}$ so that $\alpha^3 = 2$.

7. Does there exist a number $\alpha \in \mathbb{C}$ so that $\alpha(1+I, 2, 2+2I, 3-2I) = (2, 2-2I, 4, 1-5I)$?

8. Let V be a vector space over a field F. Prove that

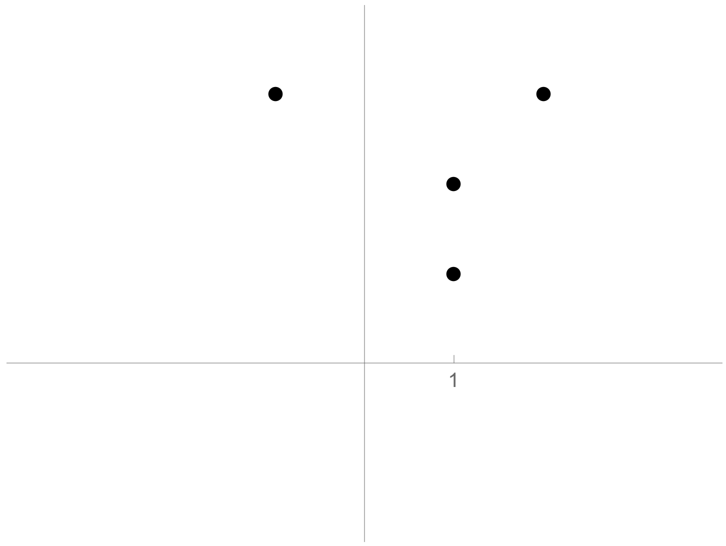
(a) For all $v \in V$, $0v = 0$.

Note: the zero on the left is the zero scalar in F and the zero on the right is the zero vector in V.

(b) For all $v \in V$, $(-1)v = -v$.

Note: the -1 on the left is a scalar in the field F, the $-v$ on the right is the additive inverse of the vector $v \in V$.

9. Using the correspondence $a+bi \longleftrightarrow (a,b)$ complex numbers can be identified with points in the Cartesian plane. The four points pictured below correspond to $z, w, z+w$, and zw for two complex numbers $z, w \in \mathbb{C}$. Which are which?



10. Consider the vector space $\mathbb{R}^4$. Which of the following subsets are subspaces?

(a) $\{(a,b,c,d) \in \mathbb{R}^4 : a+b+c=0\}$

(b) $\{(a,b,c,d) \in \mathbb{R}^4 : abc=0\}$

(c) $\{(a,b,c,d) \in \mathbb{R}^4 : a \geq 0\}$

(d) $\{(a,b,c,d) \in \mathbb{R}^4 : a=2\}$

(e) $\{(a,b,c,d) \in \mathbb{R}^4 : a=d\}$

(f) $\{(a,b,c,d) \in \mathbb{R}^4 : a+b+1=c\}$

(g) $\{(a,b,c,d) \in \mathbb{R}^4 : a+b=2c\}$

11. Consider the vector space $\mathbb{R}^{\mathbb{R}}$. Which of the following subsets are subspaces?

(a) $\{f : \mathbb{R} \to \mathbb{R} : f(1)=1\}$

(b) $\{f : \mathbb{R} \to \mathbb{R} : f(1)=0\}$

(c) $\{f : \mathbb{R} \to \mathbb{R} : f \text{ is onto}\}$

(d) $\{f : \mathbb{R} \to \mathbb{R} : f \text{ is continuous}\}$

(e) $\{f : \mathbb{R} \to \mathbb{R} : f \text{ is differentiable}\}$

(f) $\{f : \mathbb{R} \to \mathbb{R} : f''(x)=f(x)\}$

12. Let $V = \mathbb{R}^3$. Consider the following three subspaces of V

$$W = \{(0,0,a) \in V : a \in \mathbb{R}\}$$
$$X = \{(a,a,a) \in V : a \in \mathbb{R}\}$$
$$Y = \{(a,b,c) \in V : a+b+c = 0\}$$
$$Z = \{(a,a,b) \in V : a,b \in \mathbb{R}\}$$

True or False:

(a) $(1,1,-2) \in W$

(b) $(1,1,-2) \in X$

(c) $(1,1,-2) \in Y$

(d) $(1,1,-2) \in Z$

(e) W is a subspace of X

(f) W is a subspace of Y

(g) W is a subspace of Z

(h) X is a subspace of Z

(i) W is a subspace of Z

(j) $W \cap X = \{(0,0,0)\}$

(k) $X \cap Z = X$

(l) $Z = W + X$

(m) $Z = W \oplus X$

(n) $V = Y + Z$

(o) $V = Y \oplus Z$