1. Prove: If some vector in a list of vectors in a vector space V is a linear combination of the other vectors, then the list is linearly dependent.

2. Does $(1,2,3,-5),(4,5,8,3),(9,6,7,-1)$ span $\mathbb{R}^4$? Explain.

3. Is the list $(1,2,3),(4,5,8),(9,6,7),(-3,2,8)$ linearly independent in $\mathbb{R}^3$? Explain.

4. Prove that F^∞ is infinite-dimensional.

5. Suppose that v_1, v_2, v_3 is a basis for a vector space V. Prove or disprove v_1+v_2, v_1-v_2, v_3 is also a basis for V.

6. Prove or disprove: Let $p_0, p_1, \ldots, p_n$ be polynomials in $\mathcal{P}(F)$ and suppose $\deg(p_i) = i$ for $i = 0, 1, \ldots, n$. Then $p_0 p_1, \ldots, p_n$ is a basis for $\mathcal{P}_n(F)$.

7. Suppose that p_1, p_2, p_3, p_4, p_5 is a list polynomials in $\mathcal{P}_4(\mathbb{R})$ that all vanish at $x = 3$. Prove that p_1, p_2, p_3, p_4, p_5 is linearly dependent.

8. Let $U = \{(a,b,c) \in \mathbb{R}^3 : a+b+c = 0\}$.

(a) Find a basis for U.

(b) Extend your basis to a basis of $\mathbb{R}^3$.

(c) Find a subspace W of $\mathbb{R}^3$ so that $\mathbb{R}^3 = U \oplus W$.

9. Let $U = \{p \in \mathcal{P}_4(\mathbb{R}) : p(2) = p(5)\}$.

(a) Find a basis for U.

(b) Extend your basis to a basis of $\mathcal{P}_4(\mathbb{R})$

(c) Find a subspace W of $\mathcal{P}_4(\mathbb{R})$ so that $\mathcal{P}_4(\mathbb{R}) = U \oplus W$.

10. Let $U = \{p \in \mathcal{P}_4(\mathbb{R}) : \int_{-1}^{1} p = 0\}$.

(a) Find a basis for U.

(b) Extend your basis to a basis of $\mathcal{P}_4(\mathbb{R})$

(c) Find a subspace W of $\mathcal{P}_4(\mathbb{R})$ so that $\mathcal{P}_4(\mathbb{R}) = U \oplus W$.

11. Prove that any two three dimensional subspaces of $\mathbb{R}^5$ must have a nonzero vector in their intersection.

12. A function $f \in \mathbb{R}^\mathbb{R}$ is called *even* iff $f(-x) = f(x)$ for all $x \in \mathbb{R}$. A function $f \in \mathbb{R}^\mathbb{R}$ is called *odd* iff $f(-x) = -f(x)$ for all $x \in \mathbb{R}$. Check that

$$U = \{f \in \mathbb{R}^\mathbb{R} : f \text{ is even }\} \text{ and } W = \{f \in \mathbb{R}^\mathbb{R} : f \text{ is odd }\}$$

are subspaces of $\mathbb{R}^\mathbb{R}$. Prove or disprove $\mathbb{R}^\mathbb{R} = U \oplus W$.