Compute

1. Let

$$A = \begin{pmatrix} 3 & 1 & 1 & -1 & -3 \\ 0 & -2 & 1 & 0 & -3 \\ -1 & -2 & 2 & -1 & 0 \\ 3 & -1 & 1 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 0 & -2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 3 & -2 \\ -3 & 0 & -2 \end{pmatrix}.$$

Write down the matrix for AB.

2. Using the basis $(x-1),(x-2)$ for $\mathscr{P}_1(\mathbb{R})$, compute $\mathscr{M}(x-5)$, the matrix for the vector $x-5$.

3. Consider the linear map $T:\mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x,y,z) = (x-2y-2z, 2x-6y-7z, -4x+10y+10z).$$

Using the basis standard basis for the domain and the standard basis for the codomain to find $\mathscr{M}(T)$

4. Consider the linear map $T:\mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x,y,z) = (x-2y-2z, 2x-6y-7z, -4x+10y+10z).$$

Using the basis $(5,4,-2),(0,1,-1),\left(1,\frac{3}{2},-1\right)$ for the domain and the standard basis for the codomain to find $\mathscr{M}(T)$.

True/False

5. Suppose $S,T \in \mathscr{L}(V)$. If $ST = I$ then $TS = I$.

6. Suppose V is finite dimensional and $S,T \in \mathscr{L}(V)$. If $ST = I$ then $TS = I$.

7. Suppose $S,T \in \mathscr{L}(V)$. ST is invertible if and only if both S and T are invertible.

8. Suppose V is finite dimensional and $S,T \in \mathscr{L}(V)$. ST is invertible if and only if both S and T are invertible.

9. For $S:\mathbb{R}^3 \to \mathbb{R}$ given by $S(x,y,z) = x+y+z$, $\text{null}(S) = \text{span}((5,-3,-2),(0,3,-3))$.

10. The linear map $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z) = (x+y, z, x+y-z)$ is surjective.

11. The linear map $T : \mathscr{P}_1(\mathbb{R}) \to \mathbb{R}^3$ defined by $T(p(x)) = (p(1), p(2), p(3))$ is surjective.

12. Let $T : \mathscr{P}_1(\mathbb{R}) \to \mathbb{R}^3$ be the linear map defined by

$$T(p(x)) = (p(1), p(2), p(3)).$$

Using the basis $(x-1), (x-2)$ for $\mathscr{P}_1(\mathbb{R})$ and the standard basis is used for $\mathbb{R}^3$, the matrix equation $\mathscr{M}(T)\mathscr{M}(x-5) = \mathscr{M}(T(x-5))$ is the equation

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \\ -2 \end{pmatrix}$$

13. The follow homogeneous system of equations has infinitely many solutions:

$$\begin{aligned} x - 2y - 2z &= 0 \\ 2x - 6y - 7z &= 0 \\ -4x + 10y + 10z &= 0 \end{aligned}$$

14. The follow inhomogeneous system of equations has exactly one solution:

$$\begin{aligned} x - 2y - 2z &= \frac{1}{5} \\ 2x - 6y - 7z &= -17 \\ -4x + 10y + 10z &= 9 \end{aligned}$$

15. There is a linear map $T : \mathbb{R}^5 \to \mathbb{R}^2$ with

$$\text{null}(T) = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 : x_1 = 3x_2 \text{ and } x_3 = x_4 = x_5\}.$$

16. Suppose $T : V \to W$. If $v_1, \dots, v_n$ is independent in V, then $Tv_1, \dots, Tv_n$ is independent in W.

17. Suppose $T : V \to W$. If $v_1, \dots, v_n$ spans V, then $Tv_1, \dots, Tv_n$ spans the range(T).