MATH 120 In-class Activity
Day 3

Question 1. How would you draw the Venn Diagram of three **disjoint** sets?

Question 2. Let our universe $\mathcal{U}$ be the set of capital letters. That is, $\mathcal{U} = \{\mathrm{A}, \mathrm{B}, \ldots, \mathrm{Z}\}$. Consider the following sets:

$\mathcal{A}$ is the set of letters that are drawn only with lines (no curves)

$\mathcal{B}$ is the set of letters that appear **before** 'P' in the alphabet.

$\mathcal{C}$ is the set of letters in the word "DISCRETE"

(a) Write $\mathcal{A}$ in roster notation. Also write $\overline{\mathcal{A}}$ in roster notation. Are $\mathcal{A}$ and $\overline{\mathcal{A}}$ disjoint?

(b) Draw the Venn Diagram that involves the sets $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ lying in $\mathcal{U}$. Put each letter into the diagram.

(c) How would you describe the set $\mathcal{A} \cap \mathcal{B} \cap \mathcal{C}$ in words?

(d) How would you describe the set $\overline{\mathcal{A}} \cap \overline{\mathcal{B}} \cap \overline{\mathcal{C}}$ in words?

(e) List the elements in $\overline{\mathcal{B}} \cup \big(\mathcal{A} \cap \mathcal{C}\big)$. Describe this set in words.

Question 3. In this question, you will be exploring Venn Diagrams.

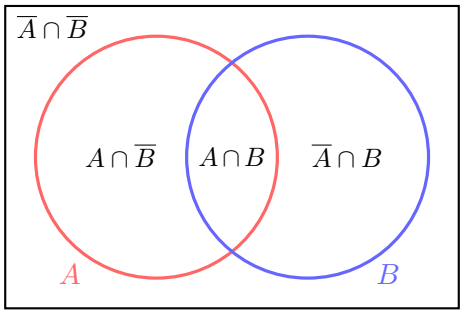
First, we set the stage by understanding essential properties of two- and three-set Venn Diagrams.

As you learned in the video lecture, Venn Diagrams are very useful as a visualization because the result of any set operations like union, intersection, and complement is represented in the diagram.

In fact, we can label **every region** of a Venn Diagram by an intersection of sets and complements of sets. For the two-set Venn Diagram involving sets A and B, the list of possible intersections is

$$\{\overline{A} \cap \overline{B}, A \cap \overline{B}, \overline{A} \cap B, A \cap B\},$$

which correspond to the regions of the Venn diagram as follows.



You can compute this list of intersections directly from the power set of $\{A, B\}$.

$$\mathcal{P}(\{A, B\}) = \{\varnothing, \{A\}, \{B\}, \{A, B\}\}.$$

The choice of whether A or $\overline{A}$ is chosen for the intersection depends on whether A appears in the corresponding element of the power set, as shown here:

Element of Power Set	A appears?	A or $\overline{A}$?	B appears?	B or $\overline{B}$?	Element of Intersection List
$\varnothing$	No	$\overline{A}$	No	$\overline{B}$	$\overline{A} \cap \overline{B}$
$\{A\}$	Yes	A	No	$\overline{B}$	$A \cap \overline{B}$
$\{B\}$	No	$\overline{A}$	Yes	B	$\overline{A} \cap B$
$\{A, B\}$	Yes	A	Yes	B	$A \cap B$

Therefore: There MUST be the same number of regions in a Venn Diagram for sets A and B as there are elements in the power set $\mathcal{P}(\{A, B\})$, which is four.

Let's check that this is also true for the three-set Venn Diagram involving sets A, B, and C. The power set $\mathcal{P}(\{A, B, C\})$ has eight elements:

$$\mathcal{P}(\{A, B, C\}) = \{\varnothing, \{A\}, \{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A, B, C\}\}.$$

Using the same correspondence, we get the list of intersections:

Element of Power Set	A?	B?	C?	Element of Intersection List
$\varnothing$	No	No	No	$\overline{A} \cap \overline{B} \cap \overline{C}$
$\{A\}$	Yes	No	No	$A \cap \overline{B} \cap \overline{C}$
$\{B\}$	No	Yes	No	$\overline{A} \cap B \cap \overline{C}$
$\{C\}$	No	No	Yes	$\overline{A} \cap \overline{B} \cap C$
$\{A, B\}$	Yes	Yes	No	$A \cap B \cap \overline{C}$
$\{A, C\}$	Yes	No	Yes	$A \cap \overline{B} \cap C$
$\{B, C\}$	No	Yes	Yes	$\overline{A} \cap B \cap C$
$\{A, B, C\}$	Yes	Yes	Yes	$A \cap B \cap C$

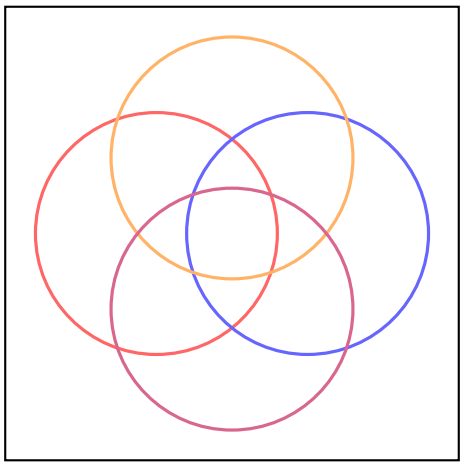
(a) Draw a Venn Diagram involving sets A, B, and C.

(b) Are there eight regions in this Venn Diagram?

(c) Place the set intersections in the correct locations of your Venn Diagram.

Now let's talk about a four-set Venn Diagram involving sets A, B, C, and D.

A mathematically incorrect picture that is often drawn for a four-set Venn Diagram is the following BAD Venn Diagram. (After class, check online - it's everywhere!!!!!)



(d) How many regions does this BAD Venn Diagram have?

(e) Using the ideas from above, determine how many regions there SHOULD be in a four-set Venn Diagram involving sets A, B, C, and D.

This means that some of the set intersections that SHOULD appear in the Venn Diagram are not appearing in this BAD Venn Diagram.

(f) Which set intersections are missing?

Now that you are aware of the problem, it's clear that a better four-set Venn Diagram is needed!

Your group's task: Draw a GOOD four-set Venn Diagram involving sets A, B, C, and D.

You will know that you have succeeded when you are able to verify that your diagram has all possible set intersections appearing **exactly once**.

During our debriefing, we will be discussing the method you used to solve this problem, so keep track of the ideas that were fruitful and the directions that were helpful in that they didn't work.