MATH 120 In-class Activity
Day 9

All but one of the following triples of counting questions has something in common in terms of **how they are solved**. Determine the common link that each triple has. Then identify the triple that does not have a common solving technique. Once you have determined the appropriate counting techniques, go ahead and answer the counting questions.

Triple 1.

1a. How many positive integer solutions are there to $x_1 + x_2 + x_3 = 20$?

1b. How many dozens of donuts can be created if there are 10 different types of donuts available?

1c. In how many ways can you choose 15 marbles out of a bag with 100 blue marbles, 100 red marbles, and 100 green marbles?

Triple 2.

2a. How many rearrangements are there of the letters in the word "uncopyrightable"?

2b. In how many ways can you pair up 8 humans and 8 pets?

2c. In how many ways can six distinct books be arranged on one shelf?

Triple 3.

3a. In how many ways can you choose three letters out of the word "junipers" to make bold?

3b. How many ten-digit sequences of 0's and 1's have the sum of their digits equal to 4?

3c. How many lattice paths leaving from $(0, 0)$ and ending at $(5, 9)$ only take East steps and North steps?

Triple 4.

4a. In how many ways can 20 distinguishable books be arranged on a bookcase with three shelves?

4b. In how many ways can 20 identical books be put into a bookcase with three shelves?

4c. In how many ways can you choose three books to remove from a shelf displaying 20 different books?

Triple 5.

5a. How many five-card hands of cards do not have all five cards of the same suit?

5b. How many subsets of $\{1, 2, \ldots, n\}$ are there with at least two elements?

5c. In how many ways can you choose a committee of size 10 with at least 2 men if there are 30 women in the club and 20 men in the club?

Triple 6.

6a. In how many ways can a game club of 50 people form a board game committee of 15 members, of which 5 are on the "Monopoly" subcommittee?

6b. How many rearrangements are there of the letters in the word "zigzagging"?

6c. How many hands of cards have three cards of one suit and two cards of a second suit?

Triple 7.

7a. In how many ways can you pick a card from a standard deck that is either a face card or is red?

7b. If there are 10 people in this class who take the bus to come to campus and 8 people who ride a bicycle to campus, and 3 people who use both modes of transportation, how many people take the bus or ride a bicycle to campus, *but not both*?

7c. How many 18-digit passwords using only letters use at least one letter from A–E and at least one letter from V–Z?

Triple 8.

8a. In how many ways can you pick a card from a standard deck that is either a face card or an Ace?

8b. How many pairs of two *distinct* integers chosen from $\{1, 2, \ldots, 101\}$ have a sum that is even?

8c. A taxi that fits at most 5 passengers must take 7 passengers to the airport in two trips. In how many ways can the passengers be broken into two groups?