

MATH 120 In-class Activity

Day 13

If you complete an activity before the end of its allotted time, work together to think about how you might push the ideas further. Alternatively, come up with different questions of a similar type to make sure you understand that learning objective more deeply.

Question 1. Let S be the set $\{0, 1, 2, 3, 4, 5\}$. Consider the function $f : S \rightarrow S$ defined by the closed formula $f(x) = |x - 3|$. Represent the function as (a) an arrow diagram, (b) in two-line notation, and (c) as a graph. Explain how you would be able to use the visualizations you created in each of (a), (b), and (c) to justify that f is a function.

Question 2. Consider the rule $n : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ where $n((x, y)) = \sqrt{x^2 + y^2}$.

(a) Give two elements in the domain of n .

(b) Apply the function n to each of those two elements.

(c) Is n a well defined function? Justify. (What is n calculating?)

(i) Show that this function n can take as input every element of the domain.

(ii) Show that this function n always outputs an element of the codomain.

(iii) Explain why this function n is unambiguous.

Question 3.

An **alphabet** \mathcal{A} is a set of elements called **letters**.

A **word** on \mathcal{A} is a sequence of letters from \mathcal{A} .

Let \mathcal{W} be the set of all words of any length on the alphabet $\mathcal{A} = \{a, b\}$.

(a) Give some elements of \mathcal{W} .

(b) Consider a rule r that takes as input a word w and searches in the word for a consecutive sequence of letters aab or bba . If it sees either one, it does the following replacement.

- Replace the consecutive string aab by the letter a
- Replace the consecutive string bba by the letter b .

Show that r is not a function.

(c) Figure out how to rewrite the rule so that it IS a well defined function.

(d) Describe the domain and codomain of this new rule.

Question 4. Come up with two rules, each of which may or may not be a well defined function. The two rules should be clear and precise. Once you come up with these rules, you will be sharing them with another group to see if they can determine which rule(s) are well defined functions. See if you can stump them!