MATH 120 In-class Activity Day 20

Question 1. Practice using sigma notation.

(a) Convert the following sigma notation expressions to expressions involving addition.

$$(i) \sum_{j=3}^{\infty} \frac{1}{j^3}$$

(ii)
$$\sum_{k=-5}^{25} \sin(k\pi)$$

(b) Convert the following expressions involving addition to sigma notation.

(i)
$$10^{100} + 10^{100} + \dots + 10^{100}$$

(ii)
$$-21 - 10 + 1 + 12 + \ldots + 111$$

(iii)
$$-1 - 3 - 9 - 27 - \ldots - 177147$$

Question 2. Practice using product notation.

(a) Convert the following product notation expressions to expressions involving multiplication.

(i)
$$\prod_{k=0}^{5} (100 + k)$$

(ii)
$$\prod_{i=2}^{\infty} \frac{(i-1)^2}{i^2+1}$$

(iii)
$$\prod_{j=m}^{n} (2j+1)$$

(b) Convert the following expressions involving multiplication to product notation.

$$(i) \ 6 \cdot 10 \cdot 14 \cdot 18 \cdot \dots \cdot 1002$$

(ii)
$$(-16) \cdot (40) \cdot (-100) \cdot (250) \cdot (-625)$$

(iii)
$$\frac{1}{4} \cdot \frac{3}{9} \cdot \frac{5}{16} \cdot \frac{7}{25} \cdot \frac{9}{36} \cdot \cdots$$

Question 3. Give a value for the following expressions, if it exists.

(i)
$$.001 + .002 + .003 + .004 + \cdots$$

(ii)
$$3+6+12+24+48+\cdots+3\cdot 2^n$$

(iii)
$$\sum_{i=1}^{10} k^2 \text{ (Careful!)}$$

(iv)
$$\sum_{i=60}^{100} i$$

$$(\mathbf{v}) \prod_{i=60}^{100} i$$

(vi)
$$\prod_{n=0}^{\infty} (-1.00001)^n$$

Question 4. Show that
$$\prod_{i=1}^{n} (2i - 1) = \frac{(2n)!}{2^n \cdot n!}$$
.

 $[\mathit{Hint 1: First understand the left hand side of the equation.}]$

[Hint 2: Try a couple of values of n to understand the pattern.]