Part I: Sets

1. Let $X = \{\{a, e, 1, 2\}, 1, 3, 4, \{i\}, o\}$. True or False:

a) True: $\{1, 3\} \subseteq X$.

b) False: $\{1, 2\} \subseteq X$

c) True: $|X| = 6$

d) False: $a \in X$

2. Let $A = \{a, b, c\}$ and $B = \{1, 2\}$.

a) List the elements of $A \times B$.

Answer. $A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$

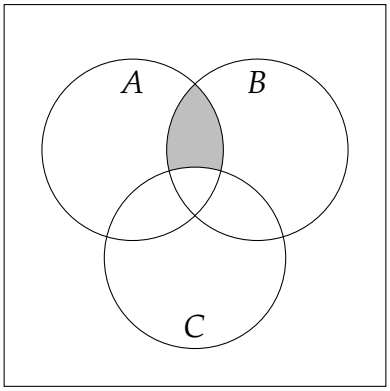
b) List the subsets of A.

Answer. There are $2^3 = 8$ subsets of A:

$$\emptyset$$
$$\{a\}, \quad \{b\}, \quad \{c\}$$
$$\{b, c\}, \quad \{a, c\}, \quad \{a, b\}$$
$$\{a, b, c\}$$

3. Draw a Venn diagram with three interacting sets A, B, and C and shade the portion of the diagram indicating $A \cap B \cap \overline{C}$.

Answer.



4. Let

$$U = \{\text{grapes, berries, bananas, oranges, apples, avocados, carrots, cherries}\}$$
$$A = \{X \in U : X \text{ has seven letters}\}$$
$$B = \{X \in U : X \text{ begins with the letter 'b'}\}$$
$$C = \{\text{grapes, apples, avocados, berries, cherries}\}$$

a) List the elements of $A \cap B$.

Answer. For the record: $A = \{berries, bananas, oranges, carrots\}$ and $B = \{berries, bananas\}$ so $A \cap B = \{berries, bananas\}$.

b) List the elements of $(A \cup B) \setminus C$.

Answer. $(A \cup B) \setminus C = \{bananas, oranges, carrots\}$.

Part II: Counting

5. How many binary words of length 10 are there?

Answer. 2^{10}

6. How many binary words of length 10 have at least two 1's?

Answer. The answer is $2^{10} - 1 - 10$.

To see that this is correct, note that there is one binary word of length 10 with no 1's, namely 0000000000 and there are ten binary words with exactly one 1:

$$1000000000,\ 0100000000,\ 0010000000, \ldots, 0000000010,\ 0000000001.$$

All the rest have at least two 1's.

7. This question relates to license plates of length 6 where each character is either a letter (A–Z) or a number (0–9).

a) How many different license plates are possible if there are no restrictions on which characters can appear where?

Answer. 36^6

b) Until recently, license plates from the Netherlands had the form XX-YY-ZZ, where each ofXX, YY, and ZZ is either a pair of numbers OR a pair of letters, like LA-97-BB or 11-92-60,or GO-MA-TH. How many different license plates of this form are possible?

Answer. There are 10^2 two digit numbers and there are 26^2 two digit letter-words. So, there are $(10^2 + 26^2)$ choices for the XX part, the YY part, and the ZZ part. This gives a total of $(10^2 + 26^2)^3$ license plates of the form described.

8. This question concerns an election in which college students are choosing a new President using Approval Voting. In this system, voters decide whether they approve or disapprove of each candidate. The candidate with the widest base of approval (the most yes votes) becomes the president. Suppose there are 15000 students voting for candidates A, B, and C and each student approves of at least one candidate.

Suppose that at the end of the campaign, Candidate A earns 10000 approval votes, Candidate B earns 9000 approval votes, and Candidate C earns 8000 approval votes. (A wins!)

Furthermore, suppose 5000 students approved of both Candidate A and Candidate B (and possibly C), 6000 students approved of both Candidate A and Candidate C (and possibly B), and 4000 students approved of both Candidate B and Candidate C (and possibly A).

a) How many students approved of all three candidates?

Answer. Let A, B, and C be the sets of students who approve of candidates A, B, and C. We have

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$
$$15000 = 10000 + 9000 + 8000 - 5000 - 6000 - 4000 + |A \cap B \cap C|.$$

So, $|A \cap B \cap C| = 15000 - 10000 - 9000 - 8000 + 5000 + 6000 + 4000 = 3000$.

b) How many students approved of candidate A only?

Answer. We compute

$$|A| - |A \cap B| - |A \cap C| + |A \cap B \cap C| = 10000 - 5000 - 6000 + 3000 = 2000.$$