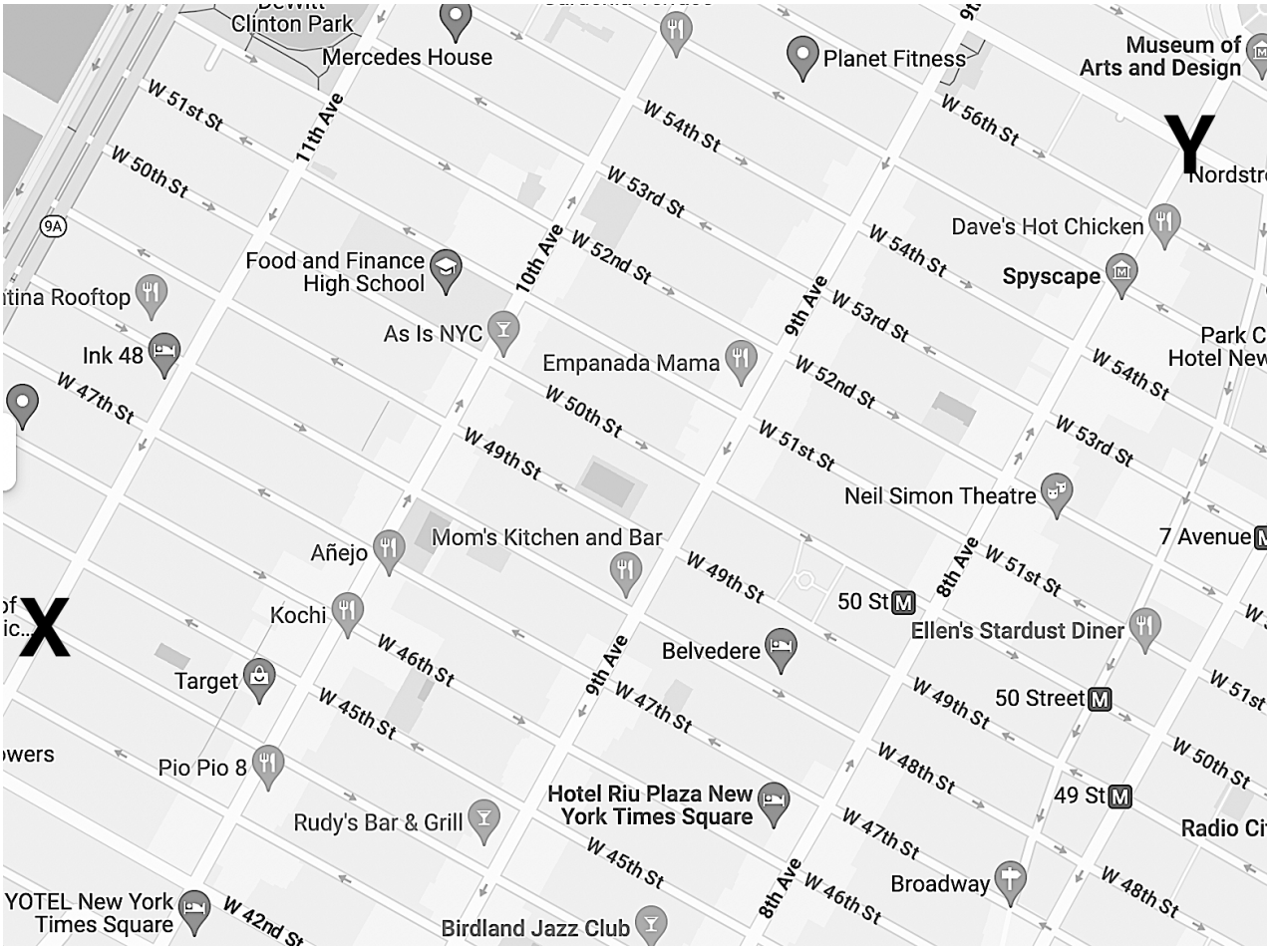
**Name:** **Midterm Exam 2** Math 120, Spring 2023

# 1 Functions and Counting

**1. [2 points]** Suppose you are trying to get from the corner of eleventh avenue and 44th street to the corner of eight avenue and 57th street (from the $X$ to the $Y$ on the map). How many different ways are there to walk there along the streets and avenues, assuming you don't go out of your way?



**2. [3 points]** Let $X = \{a, e, i, o, u\}$ and $Y = \{\text{red}, \text{green}, \text{blue}, \text{purple}, \text{yellow}, \text{orange}\}$.

*a*) How many different functions are there $X \to Y$?

*b*) How many functions $X \to Y$ are injective ?

*c*) How many functions $X \to Y$ are surjective ?

**3. [5 points]** Let $X = \{a, e, i, o, u\}$ and $Y = \{\text{red}, \text{green}, \text{blue}, \text{purple}, \text{yellow}, \text{orange}\}$ and consider $f : X \to Y$ defined by

$$a \mapsto \text{green} \qquad e \mapsto \text{green} \qquad i \mapsto \text{blue} \qquad o \mapsto \text{green} \qquad u \mapsto \text{red}$$

*a*) $f(e) =$

*b*) $f(\{e, i\}) =$

*c*) $f^{-1}\left(\{\text{red}, \text{purple}, \text{blue}\}\right) =$

*d*) $f^{-1}\left(f(\{e\})\right) =$

*e*) Find two sets $A, B \subseteq X$ for which $f(A \cap B) \neq f(A) \cap f(B)$.

# 2 Short Answer: 1 point each

**4.** What are the values of $\lfloor 41.23 \rfloor$ and $\lfloor -2.3 \rfloor$?

**5.** Define a function $F : \mathbb{N} \to \mathbb{N}$ recursively by setting $F(1) = 1$, $F(2) = 1$, and for $n \geq 2$, setting $F(n) = F(n-1) + F(n-2)$. What is $F(6)$?

**6.** What is the quotient and remainder when 57 is divided by 4?

**7.** Simplify $\dfrac{6^5}{3^6}$.

**8.** Write $\log_2(a^4) + \log_2(b^2) - \log_2(ab)$ as a single, simple expression.

**9.** Write $\log_2(703)$ using only $\log_{10}$.

**10.** Compute $\log_5\left(\dfrac{1}{5}\right) \times \log_{\frac{1}{5}}(5)$.

**11.** Simplify $\dfrac{402!}{401!}$.

**12.** True or false: A function $\{a, b, c, d\} \to \{a, b, c, d\}$ is injective if and only if it is surjective.

**13.** True or false: A function $\mathbb{N} \to \mathbb{N}$ is injective if and only if it is surjective.

# 3 Bonus

**14. [2 points]** $\lfloor \log_{10}(12345678901234567890123456789012345678901234567890) \rfloor =$

**15. [2 points]** Let $X = \{a, e, i, o, u\}$ and $Y = \{\text{red}, \text{green}, \text{blue}\}$. How many functions $X \to Y$ are surjective?