MATH 140 Second Exam Practice Test

Problem 1. The equation $x^3 - 3x^2 = a$ has 3 distinct roots exactly when

A. $a > 0$ or $a < -4$ B. $-4 < a < 0$ C. $0 < a < 4$ D. $-4 < a < 4$
E. $a > 4$ or $a < -4$

Problem 2. The skeleton of a box-shaped frame whose base is an x-by-x square and whose height is y is to be made using a piece of wire of length 16 inches. What dimensions will maximize the volume of this frame?

A. $x = 1,\ y = 2$ B. $x = \frac{3}{2},\ y = 1$ C. $x = \frac{5}{4},\ y = \frac{3}{2}$
D. $x = \frac{2}{3},\ y = \frac{8}{3}$ E. $x = \frac{4}{3},\ y = \frac{4}{3}$

Problem 3. Let f be a differentiable function and $f(0) = 2$. Suppose that we know $0 \le f'(x) \le 1$ for all $-\infty < x < +\infty$. Which of the following CANNOT be possibly true?

A. $f(1) = 2$. B. $f(2) = 4$ C. $f(-1) = 1$ D. $f(-2) = -1$
E. $f(-10) = 2$

Problem 4. What is the indefinite integral $\displaystyle\int \frac{2x}{(x^2+1)^2}\,dx$?

A. $\dfrac{x^2}{x^2+1} + C$ B. $\dfrac{-2x}{x^2+1} + C$ C. $\dfrac{-1}{x^2+1} + C$
D. Both A and B E. Both A and C

Problem 5. The average value of the function $f(x) = \sin(\pi x)$ on the interval $[0, b]$ is zero exactly when

A. b is an odd integer B. b is an even integer C. b is an integer
D. b is an odd multiple of $1/2$ E. b is an odd multiple of $\pi/2$

Problem 6. What is the shortest distance from the point $(0, 0)$ to the curve $y = \dfrac{2}{x^2}$?

A. $\sqrt{2}$ B. $\sqrt{3}$ C. $\sqrt{6}$ D. $\sqrt{8}$ E. $\sqrt{12}$

Problem 7. What is the area of the region enclosed by the graphs of $f(x) = 4 - x^2$ and $g(x) = x + 2$?

A. $\frac{9}{2}$ B. $\frac{7}{2}$ C. $\frac{5}{2}$ D. $\frac{3}{2}$ E. $\frac{1}{2}$

Problem 8. Suppose that f is a continuous function and $\int_1^9 f(x)\,dx = 6$. Then the value of $\int_1^3 xf(x^2)\,dx$ is

A. 6 B. 3 C. 2 D. 1 E. 0

Problem 9. Find the output of the following Maple statement:

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> int(1/(cos(3*u))^2,u=0..Pi/9);
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A. $\sqrt{3}$ B. $\frac{1}{\sqrt{3}}$ C. $\frac{1}{3}$ D. $\frac{1}{3\sqrt{3}}$ E. $\frac{1}{9}$

Problem 10. If h is very small, then $\tan\left(\frac{\pi}{4} + h\right)$ is approximately

A. $1 + \sqrt{2}h$ B. $\sqrt{2} + h$ C. $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}h$ D. $2h$ E. $1 + 2h$

Problem 11. If $F(x) = \int_0^{x^2} \frac{\sin t}{t+1}\,dt$, what is the derivative $F'(\sqrt{\pi})$?

A. 0 B. 1 C. π D. $\sqrt{\pi}$ E. -1

Problem 12. What is the global maximum of the function $f(x) = x^3 - x^2 - x$ on the interval $[-10, 2]$?

A. 2 B. $\frac{5}{27}$ C. 1 D. $\frac{1}{27}$ E. $\frac{1}{3}$